

# Open Research Online

---

The Open University's repository of research publications and other research outputs

## Problem solving in primary mathematics : a national survey and an in-depth analysis

### Thesis

#### How to cite:

Logan, Lindsay (2000). Problem solving in primary mathematics : a national survey and an in-depth analysis. PhD thesis The Open University.

For guidance on citations see [FAQs](#).

© 2000 The Author



<https://creativecommons.org/licenses/by-nc-nd/4.0/>

Version: Version of Record

Link(s) to article on publisher's website:

<http://dx.doi.org/doi:10.21954/ou.ro.0000e2cf>

---

Copyright and Moral Rights for the articles on this site are retained by the individual authors and/or other copyright owners. For more information on Open Research Online's data [policy](#) on reuse of materials please consult the policies page.

---

[oro.open.ac.uk](http://oro.open.ac.uk)

**Problem Solving in Primary Mathematics:  
A National Survey and an In-depth Analysis**

**Lindsay Logan, MA, MTM**

**A thesis submitted in partial fulfilment of the requirements of the  
Open University for the degree of Doctor of Philosophy .**

**September 1999**

**Northern College  
Dundee**

Author's No: P9276875  
DATE OF AWARD: 10 January 2000

## Acknowledgements

I should like to thank Northern College for providing me with facilities to conduct this research and especially to my Head of School, Brian Arnold, for arranging for time to be made available to me at critical points in the research. Thanks are also due to the Directors of Education in all 32 regions of Scotland for granting permission to invite some of their schools to take part in the National Survey. I am also indebted to Angus and Dundee Councils for allowing me access to the fourteen schools used in the in-depth study. Without the willing cooperation of the fourteen teachers and their pupils the in-depth study would not have been possible.

I should also like to extend my thanks and appreciation to my main supervisor Jennifer Tuson for her constant support, advice and encouragement. I am grateful to Ian McPherson, my second supervisor for his thought-provoking and constructive help, and to Keith Unsworth for reading and commenting on the draft thesis. Thanks also are offered to Professor John Nisbet for reading the draft thesis and offering constructive and supportive comments.

JLL

September 1999

<b>Contents</b>	<b>page</b>
Abstract	i
Acknowledgements	ii
List of Tables	vii
List of Figures	x
Chapter 1      Background and Introduction	1
1.1      Background	1
1.2      Factors influencing the choice of the areas of study	4
1.3      The components of the study	9
1.4      The structure of the thesis	10
1.5      Explanation of 5-14 Mathematics terminology	12
 Chapter 2      Literature Review	 14
2.1      Curriculum evaluation	14
2.1.1 Historical background to curriculum evaluation	14
2.1.2 The study described in terms of historical paradigms	34
2.2      Problem Solving in Primary Mathematics	40
2.2.1      General introduction	40
2.2.2      Definitions of problem solving	42
2.2.3      The importance of problem solving	47
2.2.4      Problem solving strategies	49
2.2.5      Frameworks	52
2.2.6      Metacognition	57
2.2.6.1 Definitions of metacognition	59
2.2.6.2 Teaching metacognition	62
2.2.7      Verbal reports as data	64
2.2.8      The role of affect in problem solving	70
2.2.8.1 Beliefs	72
2.2.8.2 Attitudes	73
2.2.8.3 Emotions	76
2.2.9      Collaborative group work in problem solving	76
2.2.10      Assessment and protocol analysis	82
2.2.11      Conclusions	82



Chapter 3	Research Questions	92
3.1	Introduction	92
3.2	Research questions	92
Chapter 4	Research Methods	94
4.1	The National Survey	94
4.1.1	Introduction	94
4.1.2	The pilot study	97
4.1.3	Results of the pilot study	98
4.1.4	The national survey	99
4.1.5	Summary timetable of events	102
4.2	The In-Depth Study	102
4.2.1	Introduction	102
4.2.2	An overview of the methods	104
4.2.3	Attracting participants	107
4.2.4	Briefing participating teachers	108
4.2.5	Support materials provided	110
4.2.6	Advice given to participating teachers	112
4.2.7	Groups A and B	114
4.2.8	Design of the study	118
4.2.9	Pupil observations and interviews	121
4.2.10	Pupil and teacher year-end interviews	124
4.2.11	The post-test	126
4.2.12	Item pairs analysis	126
4.2.13	Links between method and research questions	128
4.2.14	Summary timetable of events	129
Chapter 5	Results of the National Survey	130
5.1	Introduction	130
5.1.1	Summary of the findings	132
5.2	Findings of the survey	132
5.2.1	Teachers' views of the Guidelines	132
5.2.2	Effects of the Guidelines on classroom practice	136
5.2.3	Support available for teachers in implementing the Guidelines	142
5.2.4	Teachers' views on the Problem Solving and Enquiry outcome	151
5.2.5	Teachers' views on the specific issue of calculating	162
5.2.6	Use of microcomputers in school mathematics	171
5.2.7	Teachers' views on the use of context in mathematics	174
5.2.8	Teachers' own attainments in mathematics and feelings about the subject	178

5.2.9	Attitudes in schools to the Guidelines and background information	181
5.2.10	Additional comments	183
5.2.11	Conclusions	184
Chapter 6	Results of In-Depth Study	186
6.1	Introduction	186
6.2	Results of the pre- and post-tests	188
6.2.1	Results for all pupils	188
6.2.2	Comparisons of groups A and B	193
6.2.3	Analysis of school results	201
6.3	Item pairs analysis	204
6.4	Pupil interview and observation analysis	207
6.5	Year-end pupil interview results	216
6.6	Year-end teacher interview results	224
6.7	Conclusions	243
Chapter 7	Conclusions	246
7.1	Introduction	246
7.2	Research question 1	246
7.3	Research question 2	248
7.4	An evaluation of the national survey	250
7.5	Research question 3(a)	255
7.6	Research question 3(b)	256
7.7	Research question 4(a)	257
7.8	Research question 4(b)	258
7.9	Research question 4(c)	259
7.10	Research question 5	259
7.11	Research question 6	260
7.12	An evaluation of the in-depth study	261
References		
Appendices		

Word count - 85751

## **List of Appendices**

Appendix 2.1 Stake's (1967) Description Matrix

Appendix 2.2 Stake's description/judgement matrix

Appendix 2.3 Stufflebeam's CIPP model

Appendix 4.1 Questionnaire for Primary School Teachers

Appendix 4.2 Problem Solving Record Sheet (Group A schools)

Appendix 4.3 Schools used in the problem solving study

Appendix 4.4 Letter to Group A schools

Appendix 4.5 Teacher Interviews, June 1996

Appendix 4.6 Letter to Group B schools

Appendix 4.7 Problem Solving Record Sheet (Group B schools)

Appendix 5.1 National Survey Data: Cross Tabulations

Appendix 5.2 National Survey Data: Correlations

Appendix 6.1 The Pre-test

Appendix 6.2 The Post-test

<b>List of Tables</b>	<b>page</b>
Table 1.1	Outline of events in both components of the study 9
Table 2.1	Problem solving frameworks and their components 57
Table 4.1	Links between method and research questions 128
Table 5.1	Primary stages taught by teachers in the survey 131
Table 5.2	Primary stages taught 131
Table 5.3	Teachers' feelings about the Guidelines 134
Table 5.4	Effects of the Guidelines on teachers' work in mathematics 137
Table 5.5	Support given to teachers and their reactions to it 145
Table 5.6	Data from a cross tabulation of forms of support given to teachers (question 6) with teachers' views of the Guidelines (question 2) 147
Table 5.7	Teachers' feelings about the PSE outcome 153
Table 5.8	Teachers' understanding of the three aspects of PSE 154
Table 5.9	Teachers' feelings of comfort about the three aspects of PSE 154
Table 5.10	Stages reached in implementing the PSE outcome 155
Table 5.11	How pupils report on their PSE activities 156
Table 5.12	Cross tabulation of question 13 with question 14 157
Table 5.13	Changes in the modes of calculating 165
Table 5.14	Modes of calculating used by teachers themselves 165
Table 5.15	Origins of teachers' own mental methods 166
Table 5.16	The use of calculators 168
Table 5.17	Availability of computers 172
Table 5.18	Mathematics time spent on computers 172
Table 5.19	Types of software used 173
Table 5.20	Reasons given for not making greater use of computers 173
Table 5.21	Changes in teachers' use of context 177
Table 5.22	Teachers' assessments of their own abilities to teach mathematics at any level in primary schools. 179
Table 5.23	Teachers' subject teaching preferences 179
Table 5.1	Primary stages taught by teachers in the survey 181
Table 5.2	Primary stages taught 182

Table 5.24	Length of time in teaching	182
Table 5.25	Teachers within 5 years of retiral	183
Table 6.1	Mean total scores on pre- and post-tests	188
Table 6.2	Correlations of the scores on pre-test and post-test item pairs by <b>all</b> pupils	191
Table 6.3	Correlations of the scores on all item pairs on the <b>pre-test</b> by <b>all</b> pupils	191
Table 6.4	Correlations of the scores on all item pairs on the <b>post-test</b> by <b>all</b> pupils	192
Table 6.5	Mean scores for test items	194
Table 6.6	Mean facility values for test items in percentages	194
Table 6.7	Mean scores on pre- and post-tests	195
Table 6.8	Frequencies of differences between post-test and pre-test scores of pupils in groups A and B	196
Table 6.9	Scores on both tests, of pupils scoring < 8 in the pre-test Group A schools	197
Table 6.10	Scores on both tests, of pupils scoring < 8 in the pre-test Group B schools	197
Table 6.11	Correlations of the scores on all item pairs by pupils in <b>group A</b> classes	200
Table 6.12	Correlations of the scores on all item pairs by pupils in <b>group B</b> classes	200
Table 6.13	School statistics	202
Table 6.14	Analysis of variance. One-way group is school	204
Table 6.15	Sample table for recording types of responses given by two pupils to two problems	210
Table 6.16	Numbers of pupil responses in categories 1, 2 and 3. Term 1	211
Table 6.17	Observed frequencies occurring in each category 1, 2 and 3 by both groups of pupils. Term 1	212
Table 6.18	The performances of all pupils on problems 1 and 2	213
Table 6.19	Numbers of pupil responses in categories 1, 2 and 3. Term 2	214
Table 6.20	Observed frequencies occurring in each category 1, 2 and 3 for both groups of pupils. Term 2	214
Table 6.21	Observed frequencies of group A pupils scoring 1, 2 and 3 in terms 1 and 2	215

Table 6.22	Observed frequencies of group B pupils scoring 1, 2 and 3 in terms 1 and 2	216
Table 6.23	Pupils' responses to question 1, showing what they had learned	217
Table 6.24	What pupils did when they were stuck. Responses to question 3	218
Table 6.25	What pupils do when one approach to a problem doesn't work. Responses to question 4	220
Table 6.26	Strategies reported to have been used by pupils. Responses to question 5	221
Table 6.27	What pupils found hardest about problem solving. Responses to question 6	222
Table 6.28	What, if anything, pupils liked about problem solving. Responses to question 7	222
Table 6.29	How pupils reported their problem solving methods of solution. Responses to question 8	223

<b>List of Figures</b>		<b>page</b>
Figure 2.1	Schoenfeld's graph of typical 'novice' problem solving behaviour	85
Figure 2.2	Schoenfeld's graph of typical 'expert' problem solving behaviour	86
Figure 4.1	Cohen and Manion's quasi-experimental design diagram	115
Figure 4.2	Modified quasi-experimental model used in the in-depth study	116
Figure 4.3	Graph showing pupil observations and intervention points	122
Figure 6.1	Scores of all pupils on the pre- and post-tests	189
Figure 6.2	The distribution of all pupils' scores on the pre- and post-tests	190
Figure 6.3	Distribution of group A scores on the pre- and post-tests	199
Figure 6.4	Distribution of group B scores on the pre- and post-tests	199

# Chapter 1 Background and Introduction

## 1.1 Background

In 1987 the Scottish Education Department (SED) published *Curriculum and Assessment in Scotland : A Policy for the 90s*. This paper referred to the Secretary of State's intention to review existing curricular guidance and to invite the Scottish Consultative Committee on the Curriculum (SCCC), in light of that review, "to develop guidelines covering all aspects of the curriculum for the ages 5-14" (SED, 1987, p12).

The need for such a broad curricular review grew out of a belief that there were weaknesses in the existing provision. The main weaknesses identified were:

- a lack of clear statements of policy and planning in many schools
- a lack of definition in the curriculum and a corresponding uncertainty about what was to be taught at each stage
- many pupils were being insufficiently challenged at the P6/7 stage
- a degree of curricular discontinuity from P6 to S2 and a significant difference in the curricula of primary and secondary schools
- an inconsistency of approach to assessment
- poor communication with parents.

The identification of these weaknesses suggested a need for action which the paper went on to define in four broad areas:

- a clearer definition than in the past of the content and the objectives of the curriculum
- the establishment and implementation of satisfactory assessment policies in all schools



- better communication between schools and parents on the curriculum and assessment policies and practices of the school and better reporting on the progress of pupils
- consistent application in schools of the nationally agreed approach to curriculum and assessment matters.

The 5-14 development programme was set up to attend to all of these matters and in 1991 the first two sets of 5-14 national guidelines were produced for English Language and Mathematics. These were quickly followed by guidelines for other curricular areas and for Assessment and Reporting. By the time the final publication was produced, *5-14: A Practical Guide* (SOED, 1994) there were thirteen sets of guidelines. Each set was produced by a group of people, referred to as a Review and Development Group (RDG), chosen by the Scottish Education Department, and consisting of teachers, advisers, HMII and lecturers. The first task for each of these groups was to produce a draft version of their guidelines, in the form of a working paper, for national circulation and consultation.

The draft version of the mathematics guidelines was published in May 1990 entitled, *Curriculum and Assessment in Scotland. A policy for the 90s. Working Paper No 3: Mathematics 5-14* (SED, 1990). This paper was sent to all primary and secondary schools in Scotland with invitations to all local education authorities, teachers and others to comment on it within a timescale of six months. Comments received were then considered by the mathematics Review and Development Group (RDG 2), and in August 1991 the *National Guidelines: Mathematics 5-14* were published by the Scottish Office Education Department (SOED). In the rest of this report this document will be referred to in its abbreviated form as the *Guidelines*.

Schools were invited to use the *Guidelines* to "structure and develop their courses and improve the quality of learning and teaching of Mathematics" (SOED 1991, p.i). The process of implementation of the *Guidelines* was to begin in session 1991-92. Having been a member of the Review and Development Group (RDG 2), which produced the

*Guidelines*, the researcher was aware that no formal evaluation of them had been planned by the Scottish Office Education Department. Although teachers had been given an opportunity to comment on the earlier draft version (SED, 1990), the researcher believed that a study of teachers' reactions and views would be informative, after they had worked with the *Guidelines* for a number of years. The researcher was especially interested in teachers' reactions to the introduction of problem solving and enquiry as a component of the primary mathematics curriculum.

These deliberations gave rise to the first of the two main focuses of the study reported here. It is an evaluation of the implementation of the mathematics *Guidelines* to study the effects they have had on primary mathematics teaching and learning, as perceived by teachers in primary schools. Although the researcher's principal interest was in teachers' views of the problem solving aspect of the *Guidelines*, it was decided to survey teachers' reactions to all of the contents of the *Guidelines*, since the results would be of interest to the wider mathematics education community and curriculum developers.

To carry out this evaluation, it was decided to conduct a survey, by questionnaire, of the views of primary teachers throughout Scotland. At the time of their publication in 1991, the Secretary of State expressed his belief that the *Guidelines* would provide,

a firm basis for coherent, progressive teaching in Mathematics in primary and early secondary education, with a good balance between the various aspects of Mathematics. Using these Guidelines schools should be able to structure and develop their courses and improve the quality of learning and teaching of mathematics. (SOED, 1991, p.i)

Like all sets of guidelines produced as part of the 5-14 Development Programme, the Mathematics ones were non-statutory and were offered as non-prescriptive advice to schools. This reflected the official belief that the curriculum delivered in any school should reflect the professionalism of the teachers in that school.

At the outset, it was anticipated that such a survey of teachers' views might identify areas relating to the implementation which warranted further study. One of these was expected

to relate to problem solving. The effects of the *Guidelines* in secondary school mathematics and the extent to which they had improved curricular continuity between primary and secondary schools were examples of other areas which might have been identified as warranting further investigation. No detailed decisions, however, were taken as to the future and ultimate direction of the study until the results of the national survey, which comprises the first stage of the study, had become available.

Problem solving was a major innovation introduced by the *Guidelines* into primary mathematics and was anticipated by the researcher as likely to be the single biggest cause of concern to most teachers. In the event, the pilot study for the national survey, the results of which were confirmed by the survey itself, showed that this was in fact the case, and a decision was taken to investigate some aspects of problem solving in primary mathematics. This became the in-depth focus of the present study.

## **1.2 Factors influencing the choice of the areas of study**

Five distinct but closely related factors have influenced the choice of the areas to be studied in depth.

The first factor, which gave rise to the original idea for this research, related to the researcher's interest in and personal involvement with the writing of the mathematics *Guidelines*. Like many curricular innovations in Scotland these *Guidelines* had been implemented without any formal review of their effects on schools and teachers, having been planned as part of the curriculum development process. A more general evaluation of the 5-14 development programme had been funded by the Scottish Office Education Department. This was conducted between 1991 and 1995 by teams of researchers from the Scottish Council for Research in Education (SCRE), Northern College, the University of Edinburgh and the University of Strathclyde. Fourteen separate reports were produced as a result of this evaluation, covering a range of aspects of the implementation of the 5-14 development programme. None of these reports, however, was aimed exclusively at

any one subject area and, as a consequence, it was the researcher's view that none of them was sufficiently focused to meet the needs of those with a special interest in mathematics education.

The researcher was a member of Review and Development Group 2 (RDG 2), which had been established by the Scottish Office Education Department, to produce the *Guidelines*. He has subsequently designed and delivered a substantial number of in-service courses about them for teachers throughout Scotland. In the process of both of these experiences it became obvious to him that there was a need for a closer scrutiny of the effects of the *Guidelines* on mathematics in primary schools than had been planned centrally and which could be achieved by the evaluation referred to above. Whilst a number of studies specific to the implementation of National Curriculum Mathematics in England and Wales (National Curriculum Council, 1990; School Curriculum and Assessment Authority, 1993) had been conducted, no such evaluation of the *Guidelines* had been conducted or proposed exclusively for primary school mathematics in Scotland. Since the implementation of the *Guidelines* had happened on a national level and seemed to have had the greatest impact on teachers in primary schools, it was considered appropriate that the survey should involve primary teachers throughout Scotland.

The second factor which affected the choice of the in-depth focus of the study, was the researcher's personal experience as a National Development Officer for 5-14 Mathematics and as a member of Review and Development Group 2 (RDG 2). One of the researcher's responsibilities as a member of RDG 2 had been to convene a sub-group charged with writing the section of the *Guidelines* on Problem Solving and Enquiry. It was in this context that the researcher first became aware of the lack of knowledge and empirical data on how children acquire problem solving skills and indeed what these skills were. None of the members of RDG 2 had any personal experience of teaching a problem solving programme in mathematics in any cohesive, progressive and structured way. This lack of experience was shared by virtually all of the mathematics education community in Scotland and, as noted by Millett and Askew (1994), also by many

teachers in England and Wales. This was partially acknowledged by one sentence in the *Guidelines* (SOED, 1991, p9) which, referring to the development of problem solving skills, said,

At present, there is insufficient research evidence or practical experience to define progression precisely and a pragmatic approach is recommended.

Whilst this was an admirably honest admission of a lack of expertise and experience on the part of RDG 2 and indeed of all teachers, it may have concealed even greater areas of uncertainty with respect to the teaching and learning of problem solving. Examples of the kinds of questions facing RDG 2 were :

- what is meant by 'problem solving' in mathematics?
- what are the skills associated with it?
- why is problem solving important?
- which problem solving strategies are to be taught and how is this to be done?

Questions such as these arose because, along with the rest of the *Guidelines*, the problem solving and enquiry section was presented to the education community in Scotland without any accompanying theoretical justification and without reference to any empirical evidence to support its inclusion in the mathematics curriculum.

As questions such as those above were being addressed by RDG 2, it became clear to the researcher that there was a need to look more closely at the literature for help, since the advice given in the *Guidelines*, exemplified by the previous quotation, did not offer teachers very much in the way of practical help in planning or teaching a problem solving programme.

The third factor which confirmed the researcher's choice of problem solving as the in-depth focus of the study was the pilot study (Logan, 1995), which noted that,

By far the most commonly identified change in terms of the content taught was the introduction of problem solving and investigative work (p 7)

and later that,

Three quarters of the sample agreed that this (Problem Solving and Enquiry) had been one of the most difficult aspects of the Guidelines for them...(p.14).

These findings confirm results from the report of the National Curriculum Council (1990) and the *Evaluation of the Implementation of National Curriculum Mathematics at Key Stages 1, 2 and 3*, (SCAA, 1993) among others, which identified this area as a major source of concern for teachers. The findings of the pilot study were confirmed by the main national survey which followed it and was part of the study reported here.

The fourth factor which influenced this choice was the researcher's recent experience of delivering staff development and in-service courses in schools throughout Scotland on the topic of the Problem Solving and Enquiry outcome of the *Guidelines*. When the *Guidelines* were first introduced in 1991 there was a huge demand for in-service courses to introduce teachers to the fundamental ideas about problem solving and the associated ways of learning and teaching. Teachers had to be introduced to problem solving processes and strategies, to the changed roles of both teachers and learners and to the related changes in attitudes needed on the parts of both teachers and learners, to encourage successful problem solving. This first phase of staff development on problem solving, which often involved teachers gaining hands-on experiences as problem solvers in workshops, to meet a variety of problems and strategies, lasted for two or three years. Once teachers had learned about problem solving strategies and the associated teaching and learning approaches, they then had to confront the arguably more difficult question of how pupils would best acquire the skills, strategies and attitudes with which they themselves had only recently become familiar. Questions arose about planning, implementing, assessing and reporting problem solving programmes and whether a hierarchy of skills existed which could illuminate discussions on issues such as these. Staff development activities centred on these issues are still continuing and are being given a degree of urgency by school inspections by HM Inspectorate who, as was noted

in the pilot study, are asking questions about progression and structured development of strategies which schools understandably are having difficulty answering.

An important component of this staff development work has been concerned with identifying teaching actions which would help to develop pupils' awareness of problem solving strategies and their abilities to describe, select and implement them.

Considerations such as these led the researcher into the literature in a search for research evidence which could not only help teachers to address these issues but also improve pupils' problem solving skills.

The literature review is the fifth factor which has informed the choice of direction for the in-depth study.

In the last twenty years a great deal of research into mathematical problem solving has been carried out, mostly, but not exclusively, in the USA. However, many commentators e.g Silver (1988), Stanic & Kilpatrick (1988) and Lester (1994), still think that there is a continuing and pressing need for more research into mathematical problem solving. For example, asserting that there has been no agreement on how problem solving can be made an integral part of the mathematics curriculum, Lester (1994, p661) argues,

Instead of being given coherent programs with clear direction, teachers have had to be satisfied with a well-intentioned mélange of story problems, lists of strategies to be taught and suggestions for classroom activities. Although we have made considerable progress during the past 25 years, there are still many issues and questions dealing with learning, instruction, and assessment that we have only begun to address in our research.

The decision to investigate the area of problem solving reinforced the researcher's earlier decision to limit the scope of the national survey to primary schools. Although serious questions remain to be answered about the influence of the *Guidelines* on secondary schools and on primary-secondary transition practices, these issues are beyond the scope of this study.

### 1.3 The components of the study

As noted previously, the study follows two stages. These are, firstly, a national survey to evaluate the effects of the *Guidelines* on the learning and teaching of primary mathematics with particular reference to problem solving. The second component, which grew out of the first one, is an in-depth study of aspects of the learning and teaching of problem solving in primary mathematics.

Table 1.1 shows a summary timetable of the main events for both components of the study.

Table 1.1  
*Outline of events in both components of the study*

<u>Year</u>	<u>months</u>	<u>National survey events</u>	<u>Problem solving study events</u>
1993	Sept. - Dec.	Planning & literature review	
1994	Jan. - Dec.	" "	
1995	Jan. - March	Pilot study conducted	
"	March-Oct.	Analysis of pilot study	
"	Nov.	Pilot study interim report published	Planning & literature review
1996	March	Pilot study report published	Selection of participants
"	June	Planning	Briefing participating teachers
"	Sept		Beginning of year's work in schools
	Nov.	National survey conducted	
1997	May	National survey report published	
"	June		Completion of school based data collection
"	Sept - present	Analysis and write-up of results	Analysis and write-up of results



## 1.4 The structure of the thesis

This section will provide an overview of the contents of this thesis.

**Chapter 2** reviews the literature relevant to both stages of this study, and hence is divided into two parts.

Section 2.1 is concerned with literature about curriculum evaluation and begins with a brief background review of work on curriculum evaluation up to the present. It then analyses the present study of the implementation of the *Guidelines* in terms of the various historical paradigms which have been discussed and attempts to show how design decisions have been informed by previously used models of curriculum evaluation.

Section 2.2 looks critically at the literature related to problem solving in mathematics, with particular reference, where possible, to primary school mathematics in order that this study can be informed by the most recent relevant research in the field, and to identify areas where needs for further research have been apparent. A brief introduction is followed by sections on each of the following:

- understandings of what problem solving is
- the importance of problem solving
- the place of strategies in problem solving
- frameworks used by researchers to model problem solving behaviour
- the importance of metacognition in problem solving
- the use, as data, of verbal reports of problem solving processes
- affect, attitudes and beliefs about problem solving
- the place of small group work in problem solving
- assessment of problem solving and protocol analysis.

**Chapter 3** illustrates the development of the research questions relating to both components of the study, following the critical analysis of the relevant literature.

**Chapter 4** deals with the research methods used in both parts of the study, in two separate sections.

Section 4.1 describes and justifies the research methods used in conducting the national survey to study the implementation of the mathematics *Guidelines*.

A brief section on the pilot study, its results and how they informed the design of the national survey, is followed by a description of the conduct of the national survey itself.

Section 4.2 explains and justifies the methods used for the problem solving study. After an initial introduction, there are brief sections on each of the following aspects of the study:

- an overview of the methods
- attracting, briefing, advising and providing support for participating teachers
- selection of the two groups A and B
- design of the study - pupil observations and interviews; pupil and teacher year-end interviews; the post-test; item pairs analysis
- timetable of events.

**Chapter 5** deals with the results of the national survey. The results of the in-depth problem solving study are in Chapter 6. Since the results of the two parts of the study are discussed in separate chapters, a decision was taken by the researcher, in the interests of avoiding fragmentation, to include comments and discussion in the same chapter as the results for each part of the study.

Section 5.1 gives an overview of the results of the survey.

Section 5.2 has eleven sub-sections, each one dealing with the findings from different parts of the questionnaire used. Each one begins with a brief summary of the findings,

followed by details of the results. A commentary on the results concludes each sub-section.

**Chapter 6** contains the results of the in-depth problem solving study. As with Chapter 5, discussion is included alongside the details of the findings. Each of the component parts of the study is dealt with separately and, for each one, the performances of two distinct pupil groups, A and B, are compared and discussed.

**Chapter 7** makes evaluative comments, observations and conclusions about the findings of the whole study and demonstrates the extent to which each of the research questions was answered. Recommendations for future research are proposed and the contributions of the study to the field of mathematics education are summarised. Implications of the study for curriculum developers, teachers and others are discussed.

## **1.5 Explanation of 5-14 Mathematics terminology**

This section will explain some of the terminology which will be used throughout the report and which relates specifically to the *National Guidelines: Mathematics 5-14*. In these *Guidelines*, mathematics is described in terms of four broad areas of achievement which can be considered as the four component parts of the mathematics curriculum. These are referred to in the *Guidelines* as **attainment outcomes** and are currently referred to by Scottish teachers as simply **outcomes**. The four outcomes defined are:

- problem solving and enquiry
- information handling
- number, money and measurement
- shape, position and movement.

The latter three of these outcomes are further subdivided into a number of **strands**. The information handling outcome, for example, has four strands, each of which represents a line of progression with common features. These are:

- collect
- organise
- display
- interpret.

Each strand is also subdivided into short statements of attainment which are referred to as **targets**. These targets, which are described as statements of minimum competence, are defined at five broad **levels** of attainment designated as **levels A-E**. The following statement is an example of a **target** at **level B** within the 'measure and estimate' **strand** of the 'number, money and measurement' **outcome**:

Read scales on measuring devices to the nearest graduation, where each graduation is labelled. (SOED, 1991 p.34)

The five levels are defined (SOED, 1991 p.10) as:

Level A	should be attainable in the course of P1-P3 by almost all pupils.
Level B	should be attainable by some pupils in P3 or even earlier, but certainly by most in P4.
Level C	should be attainable in the course of P4-P6 by most pupils.
Level D	should be attainable by some pupils in P5-P6 or even earlier, but certainly by most in P7.
Level E	should be attainable by some pupils in P7-S1 but certainly by most in S2.

**Note:** The abbreviations P1-P7 refer to 'Primary 1 to Primary 7' - years 1 to 7 of Scottish primary schools, and S1-2 refers to the first two years of secondary schools.

The language and ideas of strands, targets and levels are not used in the problem solving and enquiry outcome because of the difficulty of defining attainment and progression within this outcome. The implications of this will be discussed in Chapter 2 and will be referred to again in Chapters 4 and 6.

### 2.1 Curriculum Evaluation

#### 2.1.1 Historical background to curriculum evaluation

Since this research is concerned with studying the effects of a particular curriculum innovation it seems appropriate that consideration should be given to historical precedents in the field of the evaluation of curriculum innovations. The design and methodology of the investigation can then benefit from whichever historical paradigms, or parts thereof, are best suited to the present task. To do this it would seem appropriate to look firstly at historical attempts to develop curricula in planned and systematic ways before considering briefly various trends and models used in the comparatively short history of curriculum evaluation.

The person who first addressed the issue of developing a curriculum in a systematic and scientific way was Dewey, whose 1902 publication, *The Child and the Curriculum*, provided this century's first influential philosophical considerations of curriculum development. He showed his support for the progressive movement in education by his emphasis on the child as an individual and the need for curriculum development to take into account the needs of children as individuals. As early as 1915 (Dewey & Dewey) he was promoting ideas such as differentiation and the value of children being given more control in the planning of their own educational experiences - ideas which are totally familiar today, even though some, such as Eisner (1985, p13), have reservations about their implementation, seeing them as, "persuasive and insightful, but.....not easily translatable into practical terms".

Child-centred concepts of education, with their stress on biological metaphors of growth and development (e.g. Harrap, 1937), were challenged increasingly from the 1920s by a

more 'scientific' approach, greatly influenced by the work of Taylor, who had been successful in improving the efficiency of the steel industry in the United States through the applications of scientific management principles such as time and motion studies, now known as Taylorism.

Taylor's scientific management approach was applied to education by Bobbitt. The latter's 1924 book *How to Make a Curriculum* set out to demonstrate how a curriculum should be a product of a systematic study of society and that the common skills needed to live a socially useful life in contemporary society, should constitute the general curriculum which would best serve that society. From this study of society would emerge a set of curricular objectives. In this respect Bobbitt anticipated the work of Tyler and Bloom in that all three tried to provide a system for dealing with the complexities of curriculum construction which relied on the use of behavioural objectives. Bobbitt's system, unfortunately, needed hundreds of objectives and was, as a consequence, extremely unwieldy. However, his rational and systematic approach, which brought a social orientation to curriculum theory, has convinced many commentators such as Eisner (1985) and Tellep (1989) that he deserves to be recognised as influential in applying perceived scientific principles to the practical problems of school education. In retrospect, however, Bobbitt's work is flawed in several important respects. In Eisner's view:

If Bobbitt attended to the analysis of life's duties, he neglected the logical difficulties of moving from 'is' to 'ought'. As much as he wanted to use scientific procedures to formulate curriculums, he paid little attention to the assessment of educational outcomes. In spite of the fact that he considered the curriculum building process complex, he underestimated the dynamic nature of the teacher's tasks. (Eisner, 1985, p26)

Bobbitt's cool, meticulous, detached and rather conservative approach did not find favour with the 'progressive' movement which was more or less influential in different areas of the USA and Europe during the 1930s. Where he relied on the rhetoric of the appliance of principle, science and specificity, the progressives placed more emphasis on the child and indeed popular works such as *The Child-Centered School* (Rugg & Shumaker, 1928) and *The Activity Movement* (Hissong, 1932) made no reference whatsoever to

Bobbitt's work. The ideas that he presented were, however, picked up and developed by others, among whom the first and arguably the most influential was Tyler.

Tyler is the first person whose theories of curriculum development and evaluation were given the description of 'model' or 'paradigm'. His publication in 1949 of *Basic Principles of Curriculum and Instruction* was perhaps the most significant event in the relatively short history of curriculum evaluation and is generally credited with launching what Hamilton (1976) refers to as the 'curriculum reform phenomenon'. The key emphasis in Tyler's model, as in Bobbitt's, is the idea and use of instructional objectives:

The process of evaluation begins with the objectives of the educational program. Since the purpose is to see how far these objectives are actually being realised, it is necessary to have evaluation procedures that will give evidence about each of the kinds of behavior implied by each of the major educational objectives. (Tyler, 1949, p110)

It should be noted at this point that Tyler, in common with his contemporaries, did not distinguish between 'evaluation' of curricula or programmes of instruction and 'assessment' of learners. He used the word 'evaluation' to cover both. It is still common practice for American writers to conflate 'evaluation' and 'assessment'. The practice in the UK, of course, is to use assessment of learners and their work for evidence in evaluating the planning and implementation of lessons, programmes and curricula.

This evaluation procedure described above consisted, in Tyler's view, of a number of essential steps:

1. Establish what the agreed aims of the curriculum are
2. Express these explicitly as behavioural objectives
3. Identify, devise and provide experiences which will give the learner opportunities to behave in the desired way
4. Decide on a way of recording the observed behaviour
5. Observe and record the degree to which the objectives are actually being realised
6. Adjust the experiences until the behaviour matches the objectives.

Although, as we shall see, it was based on a set of assumptions, many of which were later called into question, Tyler's technological view of curriculum evaluation was widely accepted for the next twenty years and to an extent still is. As late as 1975, Popham remained convinced of the value of Tyler's model:

the Tylerian tradition of educational evaluation has had an enormous impact on the thinking of educators regarding the conduct of educational evaluations. Even today, major evaluation projects....are firmly rooted in Tyler's conception of educational evaluation. (Popham, 1975, p23)

Tyler's model came to be known as the 'objectives', the 'classical' or the 'traditional' model or paradigm. It was one of the first in a category subsequently described by Scriven (1972a) and Reichardt and Cook (1979) as 'quantitative'. It was the single most dominant model in use throughout the 1950s and continued to be influential through much of the 1960s. It differed from previous models in that it was the first to focus on improving and refining curricula, whereas earlier attempts had tended to make normative judgements about individual students. In this sense Tyler greatly enlarged the scope of evaluation.

The next major catalyst for change in the whole field of education, especially in the United States, was the launch by the USSR of Sputnik - the first satellite in space - in 1957. The blame for America now being seen as suddenly no longer the most dominant technological society in the world was placed squarely on the shoulders of its education system. It was not surprising then that a number of new courses and curricula appeared, each enjoying substantial funding by the US Department of Education or the National Science Foundation. Each of these projects had to be evaluated and it soon became clear that the methods of evaluation available were not adequate for the tasks in hand. The problems encountered and some possible solutions to them were articulated in an article by Cronbach (1963), in which he shifted the focus of evaluation activities away from 'objectives' to 'decisions' and suggested that instead of asking about objectives and whether they were being achieved, evaluators should ask about the decision-making process. His three major recommendations were that:



- i) to be of greatest benefit to the developers, evaluation needed to focus on the decisions that developers must make whilst the development is still taking place,
- ii) evaluation which is used to bring about improvements during the development of the course is much more useful than evaluation which takes place after the development is finished,
- iii) evaluation must concern itself more with course performance characteristics than with achievements of individuals or with comparative studies.

The kind of comparative study to which Cronbach referred was exemplified by the evaluation of one of the biggest research projects in the UK - the introduction of the 'initial teaching alphabet' (i.t.a.) and the comparison with 'traditional orthography' (t.o.) which took place in the early 1960s. This involved over 1700 pupils and 150 teachers being divided into two groups in what would later be described (by e.g. Cohen and Manion, 1994), as a 'quasi-experimental' design. As this study evolved, a number of unforeseen difficulties appeared which Hamilton (1976) describes as 'administrative' and 'technical'. The first administrative difficulty arose because so few schools volunteered to subject their pupils to what they saw as an untried and potentially risky experiment. Secondly, it proved to be extremely difficult to provide equivalent classroom conditions for both sets of children since the initial novelty of trying out new materials and the effect of being objects of close scrutiny (i.e. the Hawthorne effect), made the experimental schools stand out as different from the control schools.

The main technical problem for the comparison was that a major variable, teacher influence, was quite uncontrolled, since teachers had been allocated to i.t.a. or t.o. according to their own choice. So differences in behaviour of the two groups of children could be attributed to this unknown variable (the teachers). Finally the three tests used to measure reading performance produced conflicting results. These examples are typical of the difficulties that dogged this and similar comparative studies.

The next major development in the field of curriculum evaluation took place as a consequence of the passing in the US Congress of the 1965 Elementary and Secondary Education Act. This act led to a number of other educational developments which, at the insistence of politicians, had to be evaluated. Once again the somewhat varied success of these evaluations produced new attempts to refine the evaluation models used.

The first and most significant of these attempts was written by Scriven (1967) in a paper which was described by Guba and Lincoln (1981, p9) as one which,

deserves to be recognised as the single most important paper on evaluation written to date.

Scriven drew the distinction between 'formative' and 'summative' evaluation and the related idea of evaluating the processes as well as the outcomes. He urged evaluators to become more judgemental, insisting that they should not only identify goals and the extent to which they were being achieved, but should also be prepared to say which of the goals were worth achieving in the first place:

if goals aren't worth achieving then it is uninteresting how well they are achieved.  
(Scriven, 1967, p52)

This statement anticipated his subsequent and perhaps better known paper of 1973 in which he espoused his famous 'Goal-Free' model. That Scriven's 'Goal Free' paper created a major impact in the field was attested to by the fact that Popham (1975) identified it as one of the two which signalled the beginning of curriculum evaluation as a field of study in its own right. This assertion was endorsed by Fraser (1984) who noted that all 39 books abstracted in his own 1982 *Annotated Bibliography of Curriculum Evaluation Literature*, appeared after 1967 and that only 7 of the 174 individual papers abstracted were published prior to 1967.

The other paper similarly identified in this way by Popham was written by Stake (1967a). In his 'Countenance' model, Stake sought to widen the evaluator's role and to provide a

model for reporting ways in which different people saw the curriculum. In this sense his is the first in what later came to be referred to as examples of 'portrayal' models of evaluation. He argued firstly that any description of evaluation had to take into account both 'formal' and 'informal' methods and argued for considered use of both types. He characterised informal evaluation as being :

recognised by its dependence on casual observation, implicit goals, intuitive norms and subjective judgment.....Informal evaluation (is) of variable quality - sometimes penetrating and insightful, sometimes superficial and distorted. (Jenkins 1976, p37)

Formal evaluation, on the other hand is recognised:

by its dependence on check-lists, structured visitations by peers, controlled comparisons, and standardised testing of students. (Jenkins 1976, p37)

Whilst Tyler's model was concerned with measuring the congruence between intended and observed outcomes, Stake argued that this process should be widened to include additionally a measure of the congruence between intended and observed 'antecedents' and intended and observed 'transactions'. 'Antecedents' are defined as conditions which exist prior to the learning and teaching occurrences which will determine the outcomes. The 'transactions' are the encounters of the learning and teaching process and 'outcomes' are the results of these transactions. The evaluator then had to ask three kinds of questions: those concerning the logical contingency (i.e. relationships) between intentions; those concerning the empirical contingency between outcomes and finally those about the degree of congruence between intentions and outcomes at each of the three levels. Stake's description matrix, which represents these ideas diagrammatically, is given in Appendix 2.1.

As well as drawing a distinction between informal and formal evaluation Stake also similarly distinguished between 'description' and 'judgement' and believed that the function of evaluation should be more than was shown in his description matrix. The evaluator should be prepared to handle judgement data. Unlike Scriven, who believed that

the evaluator should be prepared to make judgements, Stake proposed rather that the evaluator should process judgements made by other people. In order to facilitate this expanded model of evaluation, Stake described a 'judgement matrix' to stand alongside and complement his description matrix.

The first column of this matrix was to contain 'standards' which he referred to as 'benchmarks of performance'. These expected levels of performance would be referred to in the event that discrepancies arose between the first two columns in any level of his description matrix. In other words, they would be used when there was an unacceptably low congruence between 'intentions' and 'observations'. His 'judgements' column was intended to be used to interpret discrepancies between observed performance and standards. The two countenances of description and judgement were illustrated in his double matrix layout which is given in Appendix 2.2.

Like all other models before and since, Stake's countenance model had its strengths and shortcomings. On the one hand, he expanded Tyler's use of objectives to include those relating to contextual factors, to teachers and to other agencies. He provided a basis for evaluating objectives and for the first time proposed a focus of judgement as an integral component of evaluation. On the other hand, he provided no guidance as to how to specify standards, or to resolve conflicts between competing values when setting intentions. He rather naïvely assumed that there was a commonly agreed set of values in society. He failed to suggest how to find and how to take account of unintended effects. Ultimately the most damning criticism of all was that evaluation practitioners found his 12-cell double matrix too complex and difficult to use.

Whilst Stake had continued the emphasis on objectives as a focus for evaluation, others began to look for different ways of approaching the task. The next idea to gain prominence was one which had previously been mooted by Cronbach (1963). This was to focus on the use of decisions as the key emphasis in evaluation.

The most widely quoted decision-focused model is that referred to as the 'CIPP' model propounded by Stufflebeam in 1972. The purpose of Stufflebeam's Context-Input-Process-Product (CIPP) model was to provide a framework within which decision-makers could work. To do this he described a taxonomy of decision types, each one of which would be served by a type of evaluation designed especially for that purpose. His four types of decision were classified according to whether they pertained to 'ends' or 'means' and whether the decision related to 'intentions' or 'actualities'. Each of the decision types with its associated evaluation type and purpose is shown in Appendix 2.3

Stufflebeam's was the first model to expand the framework for evaluation to include not only objectives but also decisions. It related well to the contemporary interest in systems theory and it provided guidelines and support materials for a wide variety of applications. Its flaws were that it neglected the political nature of decision-making processes and the difficulty of the precise identification of the decision makers within organisations. Whilst the CIPP model took what Guba and Lincoln (1981) refer to as a 'synoptic' view of the decision-making process, it effectively ignored other models and did not deal with questions of values or standards. Finally, like other models before it, it proved to be very difficult to apply and expensive to administer and maintain.

By the beginning of the 1970s increasing criticisms of what had become known as 'quantitative' paradigms were being voiced. Hamilton (1976), looking back at a situation which he described as 'chaotic', articulated six major criticisms of the classical model:

1. Appraising a project against its pre-specified aims tended to direct attention away from more dynamic and idiosyncratic aspects of the programme and to concentrate on those which were more easily measured.
2. Mid-stream developments were discouraged in the interests of controlled experimentation. Hence the developer's and the evaluator's aims pulled in opposite directions.

3. Curricula were evaluated against questions posed by administrators and researchers rather than by the users (teachers and students).
4. Attention was focused entirely on intended outcomes. No attention was devoted to latent, unintended or unanticipated consequences which might nonetheless have been crucial to the impact of the innovation.
5. There was, according to Hamilton, never any consensus or agreement on what the intended aims, outcomes and criteria should be. These were either unmeasurable (e.g. 'the development of the whole child') or were an over-narrow reflection of the programme's aims (e.g. 'pupils will be able to wire a plug').
6. The classical model was unsuited to the gradual evolution and diversification of curriculum development. This was underlined in 1968 when the rejection by the Humanities Curriculum Project of a comparative or objectives approach signalled a clear need for a radical alternative.

These and similar criticisms prompted some fundamental changes to what had become known variously as the 'classical', 'objectives-based', or 'traditional' models of curriculum evaluation. Parlett and Hamilton (1976) introduced the new designation of the 'Agricultural-Botany Paradigm' to refer to these preceding models. This was because, according to these models:

Students - rather like plants - are given pre-tests (the seedlings are weighed and measured) and then submitted to different experiences (treatment conditions). Subsequently, after a period of time, their attainment (growth or yield) is measured to indicate the relative efficiency of the methods (fertilizers) used. (Parlett & Hamilton, 1976 pp85-86)

Parlett and Hamilton called their new model 'illuminative evaluation' and described it as belonging firmly within what they referred to as the 'social-anthropology paradigm'. This paradigm related closely to one family of approaches to research current in the fields of social anthropology, sociology and some psychiatry as well as in philosophy, and was intended to take account of the wider contexts of educational innovation. Its primary concern was to describe and interpret, rather than to measure and predict. Its aims, according to the authors, were:

to study the innovative project: how it operates; how it is influenced by the various school situations in which it is applied; what those directly concerned regard as its advantages and disadvantages; and how students' intellectual tasks and academic experiences are most affected. It aims to discover and document what it is like to be participating in the scheme, whether as teacher or pupil; and, in addition, to discern and discuss the innovation's most significant features, recurring concomitants, and critical processes. (Parlett & Hamilton, 1976, p89)

To clarify this new paradigm, Parlett and Hamilton felt obliged to draw attention to the two ideas of 'the instructional system' and 'the learning milieu'. Instructional systems which consist typically of course descriptions, syllabuses, details of methods and resources and so on, were treated in traditional models as constant and invariant. The huge variations in learning milieux in which innovations took place tended to be ignored in these models. This was a problem which they felt was addressed by the illuminative evaluation model, which took account of the fact that innovatory projects were not self-contained and independent systems which could be separated from the learning milieu of which they were part.

Illuminative evaluation was described as having three stages: investigators observe, they inquire further, then seek to explain, using data from four areas - observation, interviews, questionnaires and tests, and background sources. This approach to evaluation did not use a standard package nor slavishly follow any one research dogma, but was rather a general research strategy which was intended to be flexible and adaptable to the needs of particular cases.

Illuminative evaluation thus concentrates on the information-gathering rather than the decision-making component of evaluation. The task is to provide a comprehensive understanding of the complex reality (or realities) surrounding the project: in short, to 'illuminate'. (Parlett & Hamilton, 1976, p99)

Although qualitative models of evaluation as they emerged in the early 1970s advanced the development of curriculum evaluation in a number of ways, they were, predictably, not without their critics. The person who was most influential in effecting a further development in the field was Scriven, who has previously been referred to with respect to his 1967 paper, *The Methodology of Evaluation*.

Scriven had been one of a group of evaluators chosen to screen a number of educational innovations and developments for the Educational Testing Service in the early 1970s in the U.S. In reporting on this experience, he noted that the group's natural starting point had been to examine the objectives for which each innovation had been designed. Judgements about the products would then, in the traditional sense, depend on the extent to which the objectives had been achieved. However, it soon became apparent to him that many of the innovations they had looked at had a number of beneficial side-effects which did not coincide with the preordained objectives. In fact there were cases where the stated objectives had not been attained at all and yet where the number of useful side-effects easily justified their recommendation by the group.

This led Scriven to propose his Goal-Free Evaluation (GFE) model, not as a radical alternative to what he called Goal-Based Evaluation (GBE) but rather as a supplement to it. He argued that using objectives as organisers gave a blinkered view of the innovation, with the frequent result that beneficial but unprespecified outcomes were neglected:

It's the evaluator's job to look out for effects the experimenter (or producer, etc.) did not expect or notice. The so-called side-effects, whether good or bad, often wholly determine the outcome of the evaluation. It's absolutely irrelevant to the evaluator whether these are side or main effects: that language refers to the intentions of the producer and the evaluator isn't evaluating intentions but achievements. In fact it's risky to hear even general descriptions of the intentions, because it focuses your attention away from the side-effects and tends to make you overlook or down-weigh them. (Scriven 1973, p321)

Goal-Free Evaluation then, was similar to its precursor (Scriven's 1967 paper) in that both were interested in unintended outcomes or in not being misled by what Scriven called the 'rhetoric of intent'. Where it differed from previous models was in its emphasis on judgements, which would be made after the evaluator had matched the actual effects against an analysis of needs:

Evaluation research must produce as a conclusion exactly the kind of statement that social scientists have for years been taught is illegitimate: a judgement of value, worth or merit. (Scriven, 1974 p4)



He argued that goals were necessary for planning and implementing an innovation but that they were not sufficient and not always needed for evaluation, which considered the merit and worth of what was achieved.

However powerful Scriven's arguments were, they have largely remained at a conceptual level in the sense that his model was hardly ever adopted or implemented for a major evaluation exercise. This is not, however, to belittle the undeniable impact GFE made on the educational evaluation community. He had shown that large scale developments should always have elements of GFE, most likely in addition to those of GBE. GFE was useful, if not absolutely necessary, when a programme had few, if any, clearly defined objectives.

There were, however, criticisms. Before GFE could improve or succeed, much training of evaluators would be needed, since the model depended for its success to a large extent on the competence of evaluators using it. It also depended heavily on written evidence and on the products of the programme rather than its processes. Scriven neglected to say how the side-effects he referred to were to be identified and evaluated and gave no indication of how judgemental standards were to be derived and agreed.

In spite of these shortcomings, Scriven's model forced evaluators from then on to consider every effect - intended or not, in order to produce an optimal assessment of value. Other evaluators, whether they agreed with him or not, were obliged to take account of his views. Stufflebeam (1972) wrote:

Sponsors pay money so that certain priority needs (goals, if you will) can be met. These needs must be evaluated, and those responsible for meeting them must be judged in terms of their attempts and their achievements and failures..... Such determinations require the use of GBE although this does not diminish the desirability of GFE. (Stufflebeam, cited in Jenkins, 1976, p55)

This view that GFE had a major contribution to make in supplementing rather than replacing existing models, represented a consensus that has been widely accepted since then.

At about the same time as Parlett and Hamilton were espousing their paradigm shift, similar moves were taking place elsewhere. Papers by MacDonald (1971) and Guttentag (1971), for example, reflected a similar illuminative approach. Since the publication of his 'Countenance' paper in 1967, Stake had also increasingly emphasised what was referred to as a 'portrayal' approach to evaluation and in his *Evaluating The Arts in Education* (1975) he described his view of a 'responsive evaluation' approach as one which trades off some precision of measurement in the interests of increasing the usefulness of the findings to those in and around the programme.

Stake outlined the role and the tasks of the evaluator in an attempt to clarify further this responsive approach. The evaluator should investigate the programme from the inside by talking to people in and around it and by making personal observations about it. By discovering both its stated and real purposes and how people feel about it, he could begin to conceptualise the problems and the issues. This then allowed him to design the evaluation by specifying the data and information needed and hence to choose appropriate instruments. Data were then collected and portrayed in 'natural' ways which could take a variety of forms and formats which would be dictated by the particular case being reported and the audience for whom the report was intended.

Responsive evaluation then, like illuminative evaluation, was quite distinct from the earlier preordinate kinds. Both were at the forefront of the development of evaluations of the type which subsequently became known as 'qualitative' paradigms, although other terms such as 'process', 'humanistic', 'naturalistic' and 'transaction', have been added to the 'social anthropology' description already referred to. To these, Hamilton *et al.* (1977) added the category 'pluralist' which they defined as having application to those models which could be categorised as being:

- more extensive and not necessarily centred on numerical data
- more naturalistic, i.e. based on programme activity rather than programme intent

- more adaptable in that they were not constrained by experimental or preordinate designs.

Guba and Lincoln endorsed this approach to evaluation when they remarked that:

It is our position that responsive evaluation as proposed by Stake and elaborated by others offers the most meaningful and useful approach to performing evaluations. (Guba & Lincoln, 1981, p33)

One of those others who elaborated on Stake's responsive model of evaluation was MacDonald (1976). He espoused a model which he saw as a help to decision-makers. He felt that there should be more to evaluation than simply making judgements then passing these judgements on to the decision-makers. The exercise should be much more complex than this in that the evaluator should be aware of the total context of the development within which decision-makers have to exercise their judgement. To illustrate this, MacDonald quoted the example of an American researcher who had, subsequent to an evaluation, recommended a continuation of a state's 'bussing' policy, only to have the policy makers reject her proposal and discontinue bussing pupils from one area of the city to another in order to achieve racially integrated schools. The proposals, which had been made exclusively on educational grounds, had ignored the political realities surrounding the decision and hence were described by MacDonald as a good piece of research but an inadequate evaluation. Evaluators, he said, should provide as much information as they could to allow the decision makers to be able to make the most informed decision possible. He described his overview of evaluation in terms of three distinct types of evaluation study - bureaucratic, autocratic and democratic.

The first two types provide categories into which, according to MacDonald, all previous evaluation studies could fall, whilst the democratic model, for which he expressed a personal preference, was at that time quite new and untried.

Bureaucratic evaluation was described as a service which was provided unquestioningly by the evaluator to meet the requirements of the commissioning agency. The evaluator

had no independence and could exercise no control over the use made of his or her findings. In a sense he or she was a management consultant, hired to carry out a particular task within carefully specified parameters and as soon as the task was completed all ownership of it was surrendered by the evaluator.

Autocratic evaluation was similar to the bureaucratic model in that it too provided a service to government agencies, but differed from it in terms of the evaluator's role. In this model the evaluator was 'autocratic' in the sense that his or her contract guaranteed complete non-interference by the client and the evaluator retained ownership of the evaluation report.

The democratic model, to which MacDonald himself subscribed, was one which sought to provide information about an educational programme to a wide audience. The commissioning agency should exercise no power over the evaluator and hold no rights to his or her conclusions. Data were to be collected and reported in ways which were accessible to non-specialist audiences, so the goal of the model could be viewed as providing the greatest amount of information about a programme in an accessible form to the widest possible audience of interested observers. In this model informants and respondents were guaranteed confidentiality, thereby giving them at least partial control of the resulting data. In its concern to respect the work of participants and to provide for the needs of the educational community, the democratic model had much in common with Stake's move to 'responsive' types of evaluation.

During the latter half of the 1970s various commentators began to express some misgivings about emergent paradigms espoused by those such as Scriven, Parlett and Hamilton and their followers. Parsons, for example, agreed with the commonly expressed opinions about the shortcomings of traditional models and was in favour of many aspects of illuminative evaluation which he described as:

not merely a style of evaluation but (one which) fits more comfortably beneath the broader title of educational research. (Parsons, 1976, p127)

He did, however, go on to remark, with reference to Parlett and Hamilton's (1976) paper, that:

much remains to be done in pointing out the shortcomings and limitations of what was surely intended as a seminal paper. (Parsons, 1976, p128)

Amongst those shortcomings, Parsons (p130) listed the following:

- i) the earlier research tradition was inadequately represented since the proponents of illuminative evaluation had failed to spell out the "complexity, sophistication and rigour" of previous models and this had led to, "a misinformed view as to the facility with which this sort of research can be carried out."
- ii) the skills of the field workers had received insufficient emphasis and too little attention had been paid to the rigour and systematisation necessary in the researcher's approach,
- iii) no consideration had been given to the role of extant theories and conceptualizations and, "to enter the field in ignorance of the accumulated wealth of conceptual and theoretical schemes available is culpable".
- iv) the widened context of illuminative evaluation was still not wide enough, since it "offers only the scantiest conceptual framework in terms of the complex, interactive nature of the school situation".

Parsons concluded his commentary with the remark that whilst he viewed illuminative evaluation as very liberating, there was insufficient guidance given about how this new-found freedom was best to be used.

Similar views were expressed by Stenhouse, when he voiced reservations about standards in illuminative evaluation which he said had been:

associated with the study of cases, not of samples. Much of this work is communicated in words, but there is a lot of room in case study for a quantitative ingredient which is at present too much neglected. (Stenhouse, 1979, p6)

Stenhouse remarked that his own Scottish education had stamped him as a brake on the bandwagon of what he saw as a "breakaway, maverick and provocative" movement and went so far as to express the opinion that:

some doctoral students - and some post-doctoral researchers - are declaring themselves 'illuminative' to escape the pressure of standards and this cannot be allowed to continue. At the same time most doctoral supervisors are unsure what standards to advocate in illuminative research. (Stenhouse 1979, p7)

These remarks amount to a plea by Stenhouse for researchers not to abandon what he perceived as the academic rigour of aspects of what he called the 'psycho-statistical paradigm'.

For most of the 1970s there was an unwritten consensus that no one type of model could stand alone in meeting the needs of the wide spectrum of educational evaluations and as a consequence innovations should be judged using those elements and aspects of existing paradigms which best suited their particular needs. Some commentators interpreted this stance as reflecting the inadequacies of existing paradigms. Lewy (1973), for example, lamented the lack of balance between theoretical and empirical papers on evaluation. The person who most notably voiced his reservations about the state of the art at that time was Eisner (1977), when, in an effort to gain new insights into interpreting and evaluating what was happening in American classrooms, he used the metaphor of the art critic to introduce the two new concepts of educational connoisseurship and educational criticism. In a later publication, reflecting on the inadequacies of preceding paradigms, Eisner (1985, p91) noted:

Recognition of the assumptions, character and consequences of conventional forms of educational evaluation are insufficient to bring about change in the way we evaluate. Something more must be provided. That something more is an alternative or a complement to what now prevails, and it is the articulation and testing of this alternative at which my present work aims.

The two concepts which Eisner described and promoted owe more to the world of the arts than they do to the sciences. He refused to believe that it was possible to formulate a

set of laws which could be transformed into a prescription for good teaching just as it was impossible to prescribe how to produce a work of art.

Teaching is an activity that requires artistry..... Therefore what I believe we need to do with respect to educational evaluation is not to seek recipes to control and measure practice, but rather to enhance whatever artistry the teacher can achieve. (Eisner 1985, p91)

It was in the pursuit of this goal of enhancing the 'artistry' of teaching that he defined his 'connoisseurship' model. Connoisseurship was described as being an awareness, an understanding and a critical appreciation (though not necessarily a liking) of what had been experienced. This appreciation and awareness provide a focus for making judgements. Criticism, on the other hand, he defined as "the art of disclosure", or the ability to articulate the judgements made. In his words:

What the critic strives for is to articulate or render these ineffable qualities constituting art in a language that makes them vivid. (Eisner, 1985, p92)

This approach to evaluation was developed by Eisner with the assistance of some of his post-graduate students who spent many weeks inside classrooms observing and commenting on their observations. This kind of observation made no claims to be scientific. On the contrary, it was proudly described as non-scientific and appealed to those with a more artistic or humanistic approach to evaluation. In the past, Eisner noted, a small amount of data had been more highly valued than a large amount of insight.

Perhaps Eisner's greatest achievement was to show that scientific, or allegedly scientific, paradigms were not necessary for powerful evaluations and that the rhetoric of science was not necessary or sufficient to legitimate them. His model made a clean break from earlier ones and provided a fresh way of observing and describing educational practice. It failed, though, to suggest practical guidelines for potential users and was considered by many to be elitist in its insistence that evaluation should be carried out by highly competent connoisseurs only. Eisner himself conceded that educational criticism was not an 'efficient' method, since it took time, subtlety of perception and placed considerable

emphasis on the evaluators' writing skills. Notwithstanding these shortcomings, the connoisseurship model is generally regarded as being worthy of its place on the historical continuum and, indeed, is one of the latest evaluation models to be associated with one person.

The decade of the 1980s was a time of some turmoil and change in education in the United Kingdom as elsewhere. It was characterised by the increasing prominence of education on the platform of political and public debate. Teachers were commonly taken to task by political leaders and were often blamed for many of the real or imagined ills of the education systems or even of society in general. Teachers were variously described as incompetent, slovenly, politically biased and by implication were responsible for many of the economic ills of the previous decade. The period of teacher unrest of 1985/6 served merely to confirm in many people's minds that these (probably politically motivated) accusations had been well founded.

For at least the past decade there has been a gradual but definite shift of power away from local to central government control of education in England and Wales and to a lesser extent in Scotland. National curricular developments such as TVEI throughout the UK, the National Curriculum in England and Standard Grade and the 5-14 Development Programme in Scotland, were introduced and funded by central government. Along with the public funding went the accompanying demand for accountability. The enterprise culture prevailed and decisions began to be taken using cost analysis as a major factor, rather than measures of educational worth or merit. For most of this time central government experienced no difficulty in equating cost-effectiveness with quality of provision, or at least seeing it as a major indicator of such quality.

As a result of this clamour for greater accountability, educational evaluation in the United Kingdom underwent something of a renaissance, since demands for evaluations to be carried out at local or national level accompanied most government-funded initiatives. Nixon (1992, p4) notes that:



As a result, therefore, an upsurge in evaluation activities, involving a wide range of professional expertise, has taken place at all levels of the education system: from teachers concerned with evaluating their own classroom practice through to contracted (often university-based) evaluators focusing upon the broader aspects of policy development.

As Nixon remarked, one of the features of these evaluation developments was the increasing involvement of teachers in the process. Since much of the evaluation was examining issues of direct and daily concern to teachers, it seemed sensible to solicit their views. In arguing in favour of this trend, Nixon continued:

Those responsible for evaluating the school curriculum cannot afford to relinquish the earlier focus upon the processes of classroom interaction; but nor can they ignore the increasing stress placed upon the measurement of pre-specified outcomes. What is required is an approach to evaluation that is sensitive to that previous tradition and at the same time robust enough to respond to the heavy accountability demands currently being made upon schools. With the National Curriculum established,..... there will, it is hoped, now be an opportunity for teachers to appropriate the emergent paradigm and shape its development according to their own professional needs and values. (Nixon, 1992, pp7-8)

This involvement of teachers and the concomitant desire to give credence to their views has continued into the present generation of curriculum evaluations.

It is still true, also, that the majority of evaluations currently being undertaken owe allegiance to no particular paradigm but instead make considered use of those aspects of previous models which best meet the demands of the particular evaluation in question. The demand for evaluation in the meantime continues to be high. McNamara (1990, p223) for example, argues that there is important work to be done by outside evaluators in examining and describing the different ways in which schools have come to terms with the National Curriculum. Researchers should:

be able to demonstrate that the National Curriculum does not present the teaching profession with a non-negotiable set of constraints and given requirements. Simply by surveying and reporting on alternative practices they should be able to demonstrate that it remains possible for professional teachers to exercise their critical judgement when establishing the National Curriculum in their schools and still remain committed to their own educational values and beliefs.

In sum, the National Curriculum provides a veritable goldmine for the educational researcher who wishes to think critically and analytically about the problems of teaching and learning within busy classrooms. The researcher should be able to

provide the teacher with worthwhile information which may be pertinent to her practice and equally to make evaluative judgements about whether or not the dramatic changes which are now being wrought within the educational system, do in fact lead to improvements in the quality of both teaching and learning.

This extract from McNamara's article, subject to replacing "National Curriculum" with "the 5-14 National Guidelines", more than adequately summarises the evaluative aspirations of this research project.

### **2.1.2 The study described in terms of historical paradigms**

The first part of the study reported here will attempt to evaluate the effectiveness of the *5-14 National Guidelines in Mathematics* in Scotland and will do so by using various aspects and approaches from several of the different historical models described in the previous section. The effect of this historical review has been to convince the researcher that no one model will suffice to monitor, interpret, evaluate and describe the contents, methodologies and contexts of the *Guidelines*. As Parsons (1976, p132) explains:

Certainly it can be detrimental to the conduct of the research to enter the field in the grips of a particular theoretical model, through which attention is directed to certain issues and problems rather than allowing these to be generated through close analysis of the practical scene; but to enter the field in ignorance of the accumulated wealth of conceptual and theoretical schemes available is culpable.

Before relating this study to aspects of preceding paradigms, it is appropriate to begin by recalling the nature of this part of the study. The designation the researcher has chosen is that of 'evaluation research', a term previously used by, among others, Scriven (1974) and one which the researcher considers to reflect his belief that the study contains important elements of both evaluation and research. The term 'evaluation' has been frequently associated with the idea of commissioned studies of named curriculum projects and innovations, many of which, both here and in the U.S., have been centrally funded. Educational evaluation in this sense has been considered as a specialised branch of educational research which has grown out of a need to respond to a particular demand. In Stake's view, educational evaluation differs from educational research in that educational

evaluation focuses on a specific programme rather than on variables which are common to many programmes.

Parsons (1976, p127), on the other hand, argued that Parlett and Hamilton's model blurred the distinction between evaluation and research:

In many ways, however, illuminative evaluation is not merely a style of evaluation but fits more comfortably beneath the broad title of educational research.

This view found favour with Nisbet (1974) who described curriculum evaluation as an extension of educational research with which it shared its roots, its methods and its skills. MacDonald (1976), whilst in general agreeing with Nisbet, identified one major distinction between the two, in that the researcher tries to avoid making value judgements, whilst the evaluator is often forced into taking a political stance vis-a-vis curriculum innovations. The evaluator becomes, "embroiled in the action", whereas the researcher stands "outside the political process and values his detachment from it" (MacDonald 1976, p131). As a result, the researcher is seen here as free to define his or her own questions and to choose the processes for providing answers to them.

West (1975) agrees with MacDonald's distinction but goes further in offering other respects in which research differs from evaluation. Evaluations are normally undertaken for overtly practical purposes and their findings are normally only generalisable within the very limited confines of the project under review. Research, on the other hand, requires careful control of the experimental conditions, data gathering instruments and processes and can be constructed on a strong theoretical base.

Taking into account these distinctions as articulated by West and MacDonald, the designation of 'evaluation research' seems to be justified. The study is evaluative in the sense that it will use many established techniques and processes of successful evaluations and will seek to establish the value, in terms of worth and merit, of a major curricular innovation. It will, on the other hand, share the characteristic of research

identified by West and MacDonald (1976, p131) as being "outside the political process", in the sense that the researcher has been free to choose the questions which will be asked and to identify sources and methods of data collection without in any way being subject to outside pressures or influences.

A background feature of many evaluations carried out over the last two decades has been the demand for accountability and value for money. Sockett (1982, p7) described the function of accountability as:

an attempt to *improve* the quality of education and, it is sometimes added, to *prove* that this is being done.

This emphasis on accountability has inevitably given rise to some tensions. Nixon (1989, p92) asks:

How, within such programmes, can evaluation serve the interests both of the evaluated and of those to whom the evaluated are ultimately accountable? How, to sharpen the issue, can the evaluator be anything other than a tool of management? How can evaluation ensure that it is not compromised by the context within which it operates and without which it would barely survive?

These real concerns expressed by Nixon relate to the evaluator as someone who has been commissioned to scrutinise a particular programme. The evaluation which is the subject of this thesis does not, however, fit into this category and consequently is not subject to the political pressures felt by many project evaluators. This will be made clear to all those who are asked to participate in the research. So, whilst it is easy for the researcher to disclaim any pressures of political accountability, it is still reasonable to question the impartiality or otherwise of his political stance at the outset of the enquiry. As House (1973, p3) noted:

Contrary to common belief, evaluation is not the ultimate arbiter, delivered from pure objectivity and accepted as the final judgement. Evaluation is always derived from biased origins.

In this case the 'biased origins' relate, at least in part, to the researcher's role as a member of Review and Development Group 2 (RDG 2), a body appointed by the Scottish Office

Education Department to produce the mathematics *Guidelines*. As a consequence it seems reasonable to assume that the researcher will be less than totally impartial in his expectations for them and of course this is to an extent true. The task facing the researcher in this study then, is to attempt to develop a more impartial and critical perspective on his own beliefs and prejudices whilst at the same time benefiting from the insights and knowledge gained from being in a position as an 'insider' during the developmental phase of this particular curricular innovation. Indeed, not all of the rationale and philosophy of the *Guidelines* can be attributed to RDG 2, since some of the starting points in the production of the *Guidelines* were provided by the Scottish Office Education Department and were common to all sets of guidelines produced over the other areas of the curriculum. The views about mathematics education held by members of RDG 2 will be referred to later in Chapter 5 of this report, when the results of the survey are discussed. The researcher's primary goal, however, is to evaluate the mathematics *Guidelines* through the eyes of the practitioners - the teachers who have the responsibility for delivering and implementing this particular curricular change - in the hope that the views collected from them will provide an interesting range, or perhaps a consensus, from which important lessons for future curricular change might be drawn. In addition it is hoped that this survey of teachers' views will result in a set of judgements which are relatively free of any personal bias with which the researcher may have started out.

A common starting point for many evaluation studies was to identify the pre-specified goals or objectives of the programme, in line with traditional classical paradigms. In the present study, such an approach will be of limited value given that there were no explicit objectives identified for individual subject guidelines. There were generic aims which were articulated in the *Structure and Balance of the Curriculum* (SOED 1993), but these relate to all subject areas and whilst they may provide a focus for comment, they cannot act as a firm basis for evaluation, since they do not relate to observable outcomes in an explicit way. Given the absence of pre-specified objectives, a Tylerian approach to this study would appear to be inappropriate.

An absence of pre-stated objectives however, does not necessarily mean that there were no goals for the programme and indeed part of this study will be to examine different people's perceptions of what the programme's intentions were and to establish whether there was any consensus on the identity of these unwritten goals. Using Goal Free Evaluation ideas, the research will seek to identify actual outcomes, whether intended or not, and borrowing from Stake's approach, the study will include judgement data on these outcomes.

The established model from which this research borrows, perhaps more than most, is Parlett and Hamilton's Illuminative Evaluation which seeks, as already observed, to:

study the innovatory project; how it operates; how it is influenced by the various school situations in which it is applied; what those directly concerned regard as its advantages and disadvantages; and how students' intellectual tasks and academic experiences are most affected. (Parlett & Hamilton, 1976, p89)

Their later description of illuminative evaluation was as a general research strategy which should be adaptable and eclectic. The methods used should be in response to the demands of the particular case, rather than follow any particular research doctrine.

Equally, no method (with its own built-in limitations) is used exclusively or in isolation; different techniques are combined to throw light on a common problem. Besides viewing the problem from a number of angles this triangulation approach also facilitates the cross checking of otherwise tentative findings. (Parlett & Hamilton, 1976, p92)

Whilst these passages show clearly that elements of illuminative evaluation certainly relate to this study, there will also be similarities with Eisner's connoisseurship model which advocates the use of educational criticism to observe, describe, interpret and, to some extent, to make value judgements. This study will not only make judgements of value as advocated by Eisner and Scriven among others, but will also process, describe and analyse judgements made by others such as teachers, and advisers. In this way the approach will follow that advocated for evaluators by Stake. This will be outlined in Chapter 4 on method.

## 2.2 Problem Solving in Primary Mathematics

### 2.2.1 Introduction

This second part of Chapter 2 will discuss a number of different aspects of the teaching, learning and assessing of mathematical problem solving as they affect and inform this study. This introductory section will look at the current state of problem solving teaching and will consider references from the literature identifying aspects of the subject which deserve further study. There then will follow sections on each of the following:

Definitions of problem solving. A number of definitions and descriptions is given from a variety of sources, and a description of the author's own understanding of what is meant by a problem is given, to clarify for the reader the sense in which the word is used throughout this study.

The importance of problem solving. Some justification will be given in this section for introducing problem solving and enquiry into the primary mathematics curriculum.

Problem solving strategies. This section reviews the literature in an effort to inform the debate about problem solving strategies. Whilst there is general agreement that learners need to know about the most common problem solving strategies, there is no similar agreement on how pupils should acquire this knowledge and whether strategies should be taught explicitly or implicitly.

Frameworks for problem solving. This section looks at ways in which various researchers have attempted to devise frameworks to provide a structure for the way problem solving is planned and organised and as an aid to understanding the problem solving process. The framework chosen for use in this study is developed and justified.

Metacognition. There is a rich body of literature on metacognition in general. This section looks at the relatively smaller body of work which relates metacognition to mathematics education and discusses its place in the teaching and learning of mathematical problem solving. Different aspects of metacognition identified in the literature are discussed, along with conclusions about its important role in the problem solving process. Links will be made with the study which is the focus of this report.

Verbal reports as data. Since the study of problem solving was planned to involve taped interviews with primary pupils, it was necessary to study the references in the literature to the use of verbal reports as data. The debate about the reliability of verbal reports of children's own thought processes is discussed in some detail in this section, and conclusions are drawn as to how the lessons from the literature can be used to optimise the reliability of the verbal data used in this study.

The role of affect in problem solving. This section looks at the research evidence of how the beliefs, attitudes and emotions of both teachers and pupils can affect pupils' problem solving behaviour. Some evidence about the effects of societal beliefs, and how they too can influence the nature and context of teaching, is also adduced. The discussions in this section will inform some of the advice given to teachers involved in the study.

Collaborative group work in problem solving. It was decided to review the literature on this topic in an effort to inform the debate on which types of organisational structures were best suited for problem solving, given the absence of any relevant advice in the *Guidelines*. The issues raised here will be related to the discussions held with teachers who took part in the in-depth study described in Chapter 4.2

Assessment and protocol analysis. The need to consider the literature on this topic followed the decision to test pupils and to record interviews with both pupils and teachers as part of the in-depth problem solving study.



In the last twenty years a substantial amount of research into mathematical problem solving has been carried out, mostly in the US. Some useful comprehensive summaries and analyses have been published, most notably by Lester and Garofalo (1982), Schoenfeld (1985a), Silver (1985), Charles and Silver (1988), Lester (1994) and in England by Burkhardt (1988). In the most recent of these Lester summarised the four areas of inquiry in which progress has been 'noticeable'. These areas were, (a) determinants of problem difficulty, (b) distinctions between good and poor problem solvers, (c) attention to the teaching of problem solving and (d) the study of metacognition as an important component in the set of skills necessary for learners to become competent problem solvers.

As a result of the work of researchers such as Goldin and McLintock (1984) and Kilpatrick (1985) there is now general agreement that we are reasonably well equipped to handle area (a) above. Similarly, distinctions between good and poor problem solvers are now well established by Lesh (1985), Schoenfeld (1985a, 1987a, 1987b) and others who have done similar work. The two remaining areas are those which particularly interest the researcher since they relate, in the case of (c), to the way teachers teach, and in the case of (d), to the way children learn mathematical problem solving. The acceptance of the need for more attention to be paid to areas (c) and (d) was one of the factors which influenced the choice of the area of study for the second part of this research.

Commenting on attention to problem solving instruction, Lester (1994, p665) noted that,

About ten years ago I observed that of the several, then new, problem solving programs that had been created since 1975, none of them was firmly based in research (Lester, 1985). The situation is not so different today. What we have now are programs based largely on the folklore of mathematics teaching...

On a similar theme, Silver (1988, p186) remarked that,

teachers are faced with a popular pedagogical literature replete with suggestions for the teaching of problem solving and virtually no research base on which to support or refute the suggestions.

And finally, Stacey and Groves (1988, p206), who have done much curriculum development in Australia on problem solving, felt that,

there needs to be a dynamic interaction between research and development - at present, much of what happens in the classroom has no theory base, while at the same time many of the urgent research questions which arise from classroom practice are not being addressed.

These three extracts are typical of many which are in general agreement that there is still a definite need for research about teaching problem solving in mathematics.

Later sections of this chapter will consider the particular aspects of problem solving which have been selected as the focus of this study.

### **2.2.2 Definitions of problem solving**

In this section consideration will be given firstly to a number of attempts to provide succinct and clear definitions of problem solving. Then the thoughts of a few commentators about what they consider to be the key aspects of problem solving will be taken into account. Finally the researcher will give his own working definition of a problem which will be used in this study.

Problems are often categorised in different ways, so it seems advisable at the outset to distinguish between some of these different designations. 'Word' or 'story' problems, referred to as 'standard' problems by Askew and Wiliam (1995), are those in which the pupil must interpret the given information and choose the appropriate arithmetical operation(s) to answer the question. These can be one-, two-, or multi-step problems (Nesher and Hershkowitz, 1994), but are not the focus of this study. The problems to be considered from now on fall into the category designated by Askew and Wiliam (1995) as 'non-standard' and by Charles and Lester (1984b) as 'process' problems. These can be described as self-standing problems with one identifiable solution which can be solved

by any one of a number of different processes and in which the process itself, as well as the product or solution, can often be of interest.

An early definition of problem solving was offered by Gagné (1970, p124) who argued that,

Problem solving may be viewed as a process by which the learner discovers the combination of previously learned rules that he can apply to achieve a solution for a novel situation.

Charles and Lester (1984b, p5) defined a problem as a task which had to meet three criteria:

- the person confronting it wants or needs to find a solution
- the person has no readily available procedure for finding the solution
- the person must make an attempt to find a solution

Goldin (1982) argued that all problems could be categorised as one of four types. These are:

1. The subject does not know the answer but does possess a procedure for getting the answer, is aware that she or he possesses the correct procedure and can describe it fully.
2. Same as 1. but the subject is unable to describe the procedure in advance of carrying it out.
3. Same as 1. but the subject cannot state with certainty that he or she possesses the procedure until after the problem has been attempted.
4. The subject does not possess the procedure for solving the problem and cannot solve it successfully until extra help or advice has been given.

Goldin's four categories in fact describe reactions of solvers to problems, and it is clear that any one problem could fall into any of his categories, according to the problem solving knowledge, ability and experience of the person trying to solve it. This

categorisation does, however, reflect some classroom realities, in the sense that what may be a problem for some pupils is often not for others.

More recently Matsushita (1994, p221) said, somewhat simplistically in the researcher's view, that,

In semantic problem solving children relate objects and actions in a concrete setting to mathematical symbols and their operations.

Other commentators have attempted to describe a problem solving process or processes by listing factors or aspects which they feel are necessary components. Lester (1987), for example, listed five cognitive, non-cognitive and metacognitive factors which he believes can influence or inhibit the correct utilisation of students' knowledge. These were: knowledge, control, affects, beliefs and socio-cultural conditions. 'Knowledge' concerns facts, definitions, algorithms, heuristics and some routine but non-algorithmic procedures. 'Control' refers to using and choosing from available resources to deal with mathematical situations. It includes executive decisions about planning, evaluating, monitoring and regulating. Regulation together with knowledge about cognition constitutes 'metacognition'. 'Affects' are concerned with attitudes and emotions. Attitudes shown to have a positive effect on performance include: motivation, interest, confidence, perseverance, and willingness to take risks. Attitudes are traits of the individual, whereas emotions are situation-specific states. 'Beliefs', which are referred to as 'belief systems' by Schoenfeld (1985a), are the individual's subjective views about self, the environment, mathematics and the topic dealt with in any particular mathematical situation. Beliefs can often shape attitudes and emotions. There is evidence to suggest that children's potential for mathematical success is affected by the wealth of social and cultural influences which exist in the child's own world.

Devereux (1990) describes problem solving as a 'dynamic process', which:

- allows growth in the ability to use knowledge to solve problems
- involves discovering for oneself

- has many skills, processes and knowledge of a generalist nature that can be transferred to other situations and assist further learning
- gives the control of learning to the participant
- develops thinking skills
- aids the memory process as it involves the participant in "active" learning
- can include elements of trial and error
- involves identifying and defining the problem, devising and carrying out a plan and reviewing the results

Whilst not defining problem solving formally, several commentators such as Schoenfeld (1985a), Burkhardt (1988) and Lester (1994) have listed what they consider to be the necessary components of programmes which seek to develop problem solving skills. Four components are common to most of these lists. They are (a) frequent experience and practice of problem solving, (b) knowledge of a number of strategies, (c) metacognition and (d) appropriate beliefs and attitudes.

For teachers, it is a relatively simple task to provide frequent problem solving experiences for their pupils, given adequate resources and some ideas of differentiation and progression. The question of how, or indeed, whether to 'teach' strategies is still on the agenda for debate and research, and will be addressed more fully in Section 2.2.4. It is the researcher's belief that this question cannot be separated from the need to develop children's metacognitive skills - the focus of discussion in Section 2.2.6. The importance of beliefs and attitudes will be discussed in Section 2.2.8.

For the purposes of the in-depth study involving primary school pupils' problem solving in mathematics, which will be described fully in Chapters 4 and 6, the researcher will use, as a working description of a problem, a combination of Charles and Lester's description of 'process' problems and Askew and William's 'non-standard' designation. 'Problems' in this study will now be considered as 'self-standing, non-standard problems which have one identifiable solution, which can be solved by any one of a number of different

processes or strategies and in which the process itself, as well as the solution, can be of interest'.

Having defined the kind of problem which is the focus of this study, it is worth remembering that the problem solving and enquiry outcome of the *Guidelines* has three aspects. Pupils will be involved in problem solving and enquiry, according to the *Guidelines*, when they are:

- adopting an investigative approach to learning concepts, facts and techniques;
- working on tasks designed specifically to highlight the merits of certain approaches to mathematical thinking;
- using their mathematics in an enquiry which could be part of a cross-curricular study. (SOED, 1991, p48)

Whilst all three of these types of problem solving activities are arguably equally important, it is the second which is being referred to in this study as solving 'process' or 'non-standard' problems and which is referred to in the researcher's definition above.

### **2.2.3 The importance of problem solving**

The introduction of problem solving into the Scottish primary mathematics curriculum with the implementation of the *Guidelines* in 1991, followed similar developments in many other countries which had taken place in the previous decade. Before the 1980s, mathematics had generally been viewed as a body of knowledge consisting of facts, techniques and concepts. For example, pupils needed to know number facts relating to the four arithmetic operations. They needed to have techniques of performing measurement tasks and using arithmetic algorithms and they needed to understand a wide range of concepts related to number, measure and shape. By the beginning of the 1980s, however, this so called 'toolkit' of mathematical facts, concepts and techniques, whilst still considered necessary, was no longer seen as sufficient. Such a body of knowledge was

only good insofar as it could be used and applied to resolve a variety of situations, as noted in the well-known paragraph 249 of the Cockcroft report, *Mathematics Counts*;

The ability to solve problems is at the heart of mathematics. Mathematics is only 'useful' to the extent to which it can be applied to a particular situation and it is the ability to apply mathematics to a variety of situations to which we give the name 'problem solving'. (Cockcroft, 1982, p73)

A similar view was taken with respect to secondary mathematics in Scotland, with the publication by the Scottish Examination Board (SEB) of the *Standard Grade Arrangements in Mathematics*, in which it was noted that,

the essential aim of mathematical education is to help pupils to learn how to describe, tackle and ultimately solve problems which require the use of mathematical knowledge and techniques. (SEB, 1984, p3)

In the USA, the National Council of Supervisors of Mathematics (1977, p20) also took the view that "learning to solve problems is the principal reason for studying mathematics". In a similar vein the National Council of Teachers of Mathematics decided that problem solving should be the focus of the 1980s (NCTM, 1980). Referring to the NCTM emphasis on problem solving, Stacey and Groves (1984, p205), describing the situation in Australia, said,

Since then (1980), in almost every part of Australia, problem solving has been included formally as a major thrust in the guidelines for mathematics curricula.

Similar developments took place in a number of other countries including South Africa, Japan and New Zealand, and this emphasis on problem solving was reflected by its inclusion as one of the seven major themes at the Fifth International Congress on Mathematical Education (ICME 5) held in Adelaide, Australia in 1984.

All of these developments were influential in leading the *Guidelines* (SOED, 1991, p12) to assert that "Mathematics should be viewed in the widest sense as a problem solving activity". In England and Wales the corresponding National Curriculum Attainment Target, 'using and applying mathematics' (Ma1) was described as:

Whilst the ability to use and apply mathematics in a variety of contexts was accepted, by the end of the 1980s, as the underlying rationale for teaching problem solving by most teachers, there were some who took a wider view of problem solving. In their historical review of problem solving, Stanic and Kilpatrick (1988) identified three main themes regarding problem solving. The first was problem solving as a context - for developing new skills, for practising previously acquired skills, for recreation, for motivation and as a justification for teaching mathematics. Their second theme was of problem solving as a skill worthy of "instruction" in its own right and their third was of problem solving as art, holding that problem solving was at the heart of mathematics - a view shared with the authors of the Cockcroft report (1982), as noted previously.

These were some of the considerations which formed the background to the somewhat belated introduction of problem solving into primary mathematics in Scotland, via the *Guidelines*, which presented mathematics as, "a problem solving activity supported by a body of knowledge." (SOED, 1991, p3).

#### **2.2.4 Problem solving strategies**

In his book *Mathematical Problem Solving*, Schoenfeld (1985a) identifies five major themes which are important in developing problem solving skills. They are:

1. an emphasis on process rather than product
2. the elucidation of problem solving strategies
3. metacognition, especially the 'control' aspect
4. belief systems
5. social and school contexts.



In a similar vein, Burkhardt (1988) identifies what he refers to as the 'useful components' of problem solving. These are:

1. practice in solving problems
2. taught strategies
3. metacognition.

These are only two of many similar schemes proposed by various commentators and researchers on problem solving. As noted above, there are several recommended components of problem solving programmes which appear in the majority of such lists. These are:

1. the need for frequent practice and experience of problem solving
2. an awareness of some common strategies
3. metacognitive skills
4. appropriate beliefs and attitudes.

The first of these would probably be generally accepted by most teachers. The third and fourth are subjects of other sections in this chapter. This section will deal with the second of these components and will discuss issues relating to the teaching and learning of problem solving strategies. The study which is the focus of this report was concerned with developing pupils' problem solving strategies and investigating with teachers the most effective ways in which this development could be achieved. The contents of this section will help to formulate one of the research questions relating to problem solving strategies.

There is general agreement amongst most authorities that one of the most desirable outcomes of all problem solving programmes should be an awareness among the recipients of the programme of a number of problem solving strategies. There is, however, less agreement about how this awareness is to be achieved. Before considering the sometimes disparate views of different writers on the subject, it might be appropriate to be aware of the origins of the use of the terms 'heuristics' and 'strategies'. These are

generally attributed to Polya in his 1945 publication *How to Solve It*, in which he defines heuristic as "serving to discover" (p113) and in which he suggested a list of useful heuristic strategies. In Polya's terms heuristic reasoning related to investigative ways of working. He noted that, "We need heuristic reasoning when we construct a strict proof as we need scaffolding when we erect a building" (p113). Since then Polya's use of the term heuristic has been subject to some rather loose interpretations. Schoenfeld (1988, p10) for example, described Polya's heuristic strategies as "rules of thumb for making progress when you are stuck". Krulik and Rudnick (1988) referred to heuristics as the components of their five-step problem solving plan of a type which in Section 2.2.5 is referred to as a framework. The general tendency in recent years has been for commentators on problem solving to use the terms, 'heuristics', 'strategies' and 'heuristic strategies' interchangeably.

Two questions which reflective teachers of problem solving must address at some stage are how or indeed whether to teach problem solving strategies. The following quotations from Schoenfeld (1984, p11) might encourage some reflection on the topic.

there is a danger to the faddish addition of a section or two on 'problem solving via heuristics' to various texts, thus thinking that one has made significant progress toward teaching problem solving.

heuristic strategies .....are not ends in themselves, but rather a means to an end..... In much curriculum development, we see adjunct units on problem solving strategies, a week on this problem solving technique or that (e.g. "Here's how to find patterns"), taught and tested as a separate bit of subject matter. This kind of approach trivialises mathematical thinking. It may be better than nothing at all, but not by much.

almost universally, attempts to have students learn problem solving via heuristics were "encouraging" but not really successful.

These views seem to find support from Stacey and Groves (1988, p205) who remark that,

We do not endorse the teaching of extensive, perhaps even classified lists of strategies.

and from Lester (1994, p666) who states,

Teaching students about problem-solving strategies.....does little to improve their ability to solve mathematics problems in general.

The message from these extracts would appear to be that lists of strategies should not be taught explicitly and that teachers should guard against such an approach. This idea, though, would appear to be tempered by other passages from other researchers. The following is an illustration of an opposing view.

The explicit teaching of strategies is effective; it also provides teachers with some place for their explanatory skills. Further, it is generally safe to assume that if a strategy or technique is not taught explicitly, students will not learn it. (Burkhardt *et al.*, 1986, p214)

Stacey and Groves qualify their previous statement when they assert that,

Simple problem solving strategies, which arise naturally in a variety of contexts and whose use can clearly be seen to enhance the process, should be made explicit to students. (Stacey and Groves, 1988, p205)

Other studies have supported this view that there is a need for an explicit approach to the teaching of general or 'domain independent' strategies. By 'domain independent' is meant those strategies, such as 'draw a diagram' or 'try a simpler case', which although commonly seen to be exclusively mathematical, do have applications in other areas of study. Chinnappan and Lawson (1996) found that instruction in the use of some general strategies significantly improved the performance of pupils not only on tasks similar to those used in the training, but also in the solutions of different types of problems. Campione, Brown and Connell (1988) asserted that less able pupils would not acquire strategies unless they were given detailed and explicit instruction in their use and that the more complex the strategy, the more detailed the instruction needed to be. Similar results were found by Schoenfeld (1979), Mayer (1985) and Whitebread (1996). In spite of this seemingly persuasive body of evidence supporting the explicit teaching of strategies, Sweller (1990) argued that there was no strong evidence to support the teaching of general or domain independent strategies in mathematics classes. In particular he argued that there was no evidence that students trained in strategy use were able to transfer these strategies to situations or problems unrelated to those used in training.

Although the balance of the arguments listed above would appear to favour the direct teaching of strategies, there is by no means unanimity on the subject. Whilst it is clear that pupils must learn strategies, the means by which this learning is to be achieved remain open to debate. The ability to use strategies may depend on a range of variables such as intelligence, background knowledge, previous experience, the style of teaching and the contexts in which problem solving takes place. This suggests to the researcher that both the proponents and opponents of direct teaching of strategies could be correct in certain situations. This debate about whether strategies could or should be taught was a live issue in the selection of research questions for this study as will be seen in later chapters, in which the researcher will propose a particular approach which, it will be hypothesised, might improve pupils' knowledge of strategies. No help about the teaching of strategies was offered to teachers in the *Guidelines*, which only provided a list of commonly used problem solving strategies and illustrative examples of their use, but offered no advice as to how they should be introduced, sequenced, taught or learned.

This debate about how problem solving strategies should be taught or learned was an influential factor in the researcher's decision, described later, in Section 4.2, to establish for comparative purposes two groups of pupils whose teachers would approach the teaching of strategies in different ways.

### 2.2.5 Frameworks

Since serious attention was first paid to the teaching of problem solving as an important component of mathematics, frameworks have been devised to provide a structure for the way problem solving is planned and taught, and as an aid to analysing pupils' problem solving performance. Since teachers in this study were being asked to teach a problem solving programme for a whole year, for the first time in most cases in an organised way, the researcher felt it was important that they be given a supportive framework within which they could work and which was easy to understand and apply with their pupils.

The following discussion on the literature relating to frameworks was used to inform and justify the choice of framework suggested to teachers in this study.

The first such framework was proposed in 1945 by Polya, who is generally recognised as the father of modern mathematical problem solving. It consisted of the four components of: "understanding the problem; devising a plan; carrying out the plan; looking back" (pp. XVI-XVII)

This framework and variations of it are still used by many teachers and researchers, showing the importance of the impact it has had on the development of problem solving as an element of mathematics education in the intervening years. Although Polya did not refer explicitly to metacognition in his framework, it is likely that, implicitly at least, metacognitive behaviours were involved in implementing and linking each of these components.

Schoenfeld (1987b, 1983, 1992) describes a framework with slightly more highly specified elements than Polya's for analysing protocols of college students' problem solving work, a protocol having been defined as,

an objective record or traces of a sequence of overt actions taken by individuals in the process of solving problems. (1983, p347)

His framework has the elements, Read, Analyse, Explore, Plan, Implement and Verify and is discussed further in Section 2.2.10

Garofalo and Lester (1985) presented what they referred to as a "cognitive-metacognitive framework" (p170) which they felt was relevant for more than only problem solving tasks. It purported to specify key points where metacognitive decisions were likely to influence cognitive actions. The four categories in their framework were:-

orientation - strategic behaviour to assess and understand a problem.

organisation - planning of behaviour and choice of action.

execution - regulation of behaviour to conform to plans.

verification - evaluation of decisions made and outcomes of executed plans.

These four categories were similar to those of Polya but were more broadly defined than Polya's. This framework was intended to serve as a tool for analysing metacognitive aspects of mathematical performance which would take place at different points within the framework.

The framework described by Mason, Burton and Stacey (1985) consisted of the three interlinked phases of Entry, Attack and Review and was one of the few in the literature which, as well as being informative for teachers, was designed as an aid to help pupils think about the process of problem solving. In its simplicity and vividly metaphorical character, it contrasted sharply with others which were proposed at about the same time, such as that suggested by Adams, (1986) which contained seven elements.

Frameworks which have been proposed since then have tended to be variations or refinements of Polya's original one. Mayer's (1987) model which was used subsequently to train students and teachers in two studies conducted by Cardelle-Elawar (1992, 1995), suggested that four types of processes or knowledge are required for mathematical problem solving. They are: translation; integration; planning and monitoring; solution/execution.

'Translation' is needed for the subject to read and make sense of the words of the problem. 'Integration' requires the student to put all the component clues together and to know and recognise problem types. 'Planning and monitoring' require a knowledge of problem solving strategies to devise and monitor a solution plan. 'Solution/execution' demands the ability to use procedural knowledge to carry out any necessary calculations.

A similar Polya-based framework was suggested by Resnick (1988) in which she identified four key processes that she felt should be repeated in each new problem solving attempt. These were: 'planning'- analysing the problem to determine appropriate procedures; 'organising' the steps for a chosen procedure; 'carrying out' those steps; 'monitoring' each of the above processes to detect errors of sense and procedure.

The framework used by Artzt and Armour Thomas (1990) was an adaptation of Schoenfeld's (op. cit) which also used episodes and executive decision points. This one explicitly attached metacognitive or cognitive descriptors to each of the elements (or episodic categories) of the framework. These were: 'read' (cognitive); 'understand' (metacognitive); 'analyse' (metacognitive); 'plan' (metacognitive); 'explore' (cognitive or metacognitive); 'verify' (cognitive or metacognitive); 'watch and listen' (cognitive).

Much more recently, in the Scottish context, the *Guidelines* offered a new framework for problem solving which was described as a 'definition' of the Problem Solving and Enquiry outcome and which consisted of "three broadly interdependent steps - starting a task; doing a task; reporting on a task" (p12). In the researcher's view this would better have been described using the four steps illustrated in the diagrammatic representation which accompanies the definition and which includes the important step of 'evaluating'.

A comparison of the frameworks mentioned above and that of the *Guidelines*, illustrates some differences which are structural in nature and another which is more substantive. The structural differences are related to the different ways of describing how pupils get started with a problem. As can be seen from Table 2.1 below, most frameworks break down the "Starting a Task" phase of the *Guidelines* into two or more highly specified sub-stages. There are similar differences with respect to the "Doing" and "Evaluating" phases of the cycle as defined in the *Guidelines*. The substantive difference between their framework and the others previously mentioned is in the inclusion of a "Reporting" phase. Table 2.1 illustrates the correspondences, similarities and differences between a number of the frameworks referred to above.

Table 2.1  
*Problem solving frameworks and their components*

Frameworks	Components			
Scottish <i>Guidelines</i>	Starting	Doing	Reporting	Evaluating
Artzt et al.	Reading Understanding Analysing Planning	Exploring		Verifying Watching and listening
Resnick	Planning Organising	Carrying out		Monitoring
Mayer	Translating Integrating Planning	Solution Execution		
Mason et al.	Entry	Attack		Review
Garofalo et al.	Orientation Organisation	Execution		Verification
Schoenfeld	Reading Analysing Planning	Exploring Implementing		Verifying
Polya	Understanding the problem Devising a plan	Carrying out the plan		Looking back

As noted above, the distinctive feature of the model given in the *Guidelines* is the inclusion of a stage within the problem solving process which highlights the need for pupils to report their findings. It is for this reason that this model has been chosen as the one which will be a focus for the work done with pupils throughout the school-based work of this study. Also, because of the possibility that pupils' ability to give verbal reports on their problem solving processes may be related to their metacognitive skills, it is pertinent, at this stage, to consider the literature, firstly on metacognition, then on the use of pupils' verbal reports as data. This is done in the following two sections.

### 2.2.6 Metacognition

It is generally recognised that psychological research on what has now become known as 'metacognition' was initiated by Flavell and his co-workers in 1970 and his 1976 description of metacognition is now widely accepted:



'Metacognition' refers to one's knowledge concerning one's own cognitive processes and products or anything related to them, e.g., the learning-relevant properties of information or data.....Metacognition refers, among other things, to the active monitoring and subsequent regulation and orchestration of these processes in relation to the cognitive objects on which they bear, usually in the service of some concrete goal or objective. (Flavell 1976, p232)

At about the same time, Piaget (1976) referred to 'reflexive abstraction' to describe a mechanism for reorganising and extracting knowledge and Skemp (1979) described the ability to think closely about one's own mental processes as 'reflective intelligence' (p 175). It was Flavell's description which prevailed, however, and in the following decade there was a growing amount of research and related literature on the subject.

Little of this research, however, related directly to mathematics education in general or problem solving in particular. Schoenfeld (1982) believed that there was much more to mathematical problem solving than was at that time being modeled. One of the missing elements he identified was metacognition which he believed was the 'driving force' in cognitive performance and as a consequence much more research was needed to explore it. He stated at that time that mathematics educators could no longer afford to ignore psychological research and because of the lack of attention paid to metacognitive factors he described most "instruction" in mathematics as being, "in a very real sense, deceptive and possibly fraudulent." (p27). From his 1985a publication, *Mathematical Problem Solving* onwards, Schoenfeld advocated the inclusion of metacognition as a necessary and integral component of any problem solving programme.

At the same time Garofalo and Lester (1985) continued on the theme of a lack of systematic research on metacognition in mathematics, which they described as,

an important aspect of intellectual functioning that, until recently, has not received from the mathematics education community the share of study it deserves. (p174)

They postulated three reasons for this state of affairs. Firstly, covert mental activities, which include metacognitive ones, were seen as either notoriously difficult or impossible to observe. Following the arguments of Nisbett and Wilson (1977), many researchers

still believed that people had no introspective access to their own thought processes. Secondly, even those who did accept verbal self-reports as data, still felt that the act of verbalisation of cognitive processes might change the nature of these processes (Ericsson and Simon, 1980, Ginsburg, et al., 1983). Finally, metacognitive phenomena had been considered up till then by psychologists to be too ill-defined to be valid as objects of scientific investigation. Since mathematics educators had often followed the lead of psychologists in deciding what to study, the role of metacognition had remained largely overlooked with a few notable exceptions, (Schoenfeld, 1981, 1983; Silver 1982a, 1982b; Lester and Garofalo, 1982).

From the mid 1980s, urged on by the kind of remarks reported above, the mathematics education community began to show some more interest in the study of the role of metacognition in the development of pupils' problem solving skills. Before looking at the results of some of the research conducted since then, the following section will attempt to contribute to an understanding of the field by looking at how metacognition has been described or defined by different researchers.

#### **2.2.6.1 Definitions of metacognition**

The following are examples of other commentators' attempts to interpret Flavell's original (1976) description of metacognition.

- the awareness of one's mental processes, the capacity to reflect on how one learns, how to strengthen memory, how to tackle problems systematically - reflection, awareness, understanding and perhaps ultimately control (Nisbet and Shucksmith, 1986, p8)
- the knowledge, monitoring and control of one's own learning. (Baird, 1986, p264)
- understanding of knowledge, an understanding that can be reflected in either effective use or overt description of the knowledge in question. (Brown, 1987, p65)
- second order cognitions: thoughts about thoughts, knowledge about knowledge or reflections about actions. (Weinert, 1987, p8)
- an aspect of critical thinking that includes the student's abilities to (a) develop a systematic strategy during the act of problem solving and (b) reflect on and

evaluate the productiveness of his or her own thinking processes. (Cardelle-Elawar, 1992, p110)

- thinking about one's own thoughts. Thinking can be of what one knows (i.e. metacognitive knowledge), what one is currently doing (i.e. metacognitive skill), or what one's current cognitive or affective state is. (i.e. metacognitive experience). (Hacker, 1998, p3)

A less typical description of metacognition than those above was provided in Japan by Hirabayashi and Shigematsu (1987) who referred to children as having two egos. The first is the acting ego whilst the second is the executive ego which monitors, assesses and controls the other. This executive ego is referred to as the 'inner teacher' because it observes, criticises and controls the original self in the manner of a teacher.

In mathematical problem solving, the most commonly accepted definitions of metacognition are two which build on that first provided by Flavell. Garofalo and Lester (1985) separated Flavell's definition into its two component aspects, namely 'knowledge' and 'control and regulation' and Schoenfeld (1987b) took this description a step further by adding the third aspect of 'beliefs and intuitions'.

'Knowledge' in this context refers to what one knows about one's own thought processes and about cognitive phenomena. This aspect of metacognition has been subdivided into three categories of knowledge (a) 'person knowledge' - what one knows about oneself, including one's beliefs, and others as cognitive beings. In the particular context of mathematics this would include one's beliefs about one's mathematical ability in general and with respect to mathematical problem solving in particular. (b) 'task knowledge' - knowledge about the scope and requirements of particular tasks and why some tasks should be more difficult than others. (c) 'strategy knowledge' - knowledge of general and specific strategies and their usefulness in problem solving (Flavell and Wellman, 1977)

'Control or regulation' is the aspect of metacognition concerned with a variety of decisions and strategic activities. It is concerned with keeping track of the processes being used and using this information to inform the choice of subsequent problem

solving actions. In mathematical problem solving, examples of activities which illustrate the control or regulation aspect might be: planning courses of action; selecting certain strategies to get started with a problem; evaluating the likelihood of success of actions currently being used; deciding which new direction to follow when 'stuck'; deciding when to abandon non-productive strategies - the so-called 'impasse-based' theory discussed by Roberts and Erdos (1993). Actions such as these are among those categorised as 'managerial' decisions by Schoenfeld (1981). He also described as 'tactical' those decisions of a non-metacognitive nature, which related to areas such as choice of arithmetical operations or algorithms in attempts at solving problems.

'Beliefs and intuitions' refer to the importance attached to beliefs of students about mathematics in general or about problem solving in particular. There is a growing body of research evidence which shows the importance of the beliefs of students, teachers and society at large on problem solving performance. This is discussed further in Section 2.2.8.

#### **2.2.6.2 Teaching metacognition**

This section looks at research reports of attempts which have been made to develop children's metacognitive skills by overtly teaching them. These developments took place subsequent to the pleas for more research in this area remarked on at the beginning of this section (2.2.6). The following approaches are those which have been most often tried and which have had varying amounts of success.

Whole class discussions. In this approach the teacher focuses the pupils' attention on the need to be questioning, to decide what is relevant and to be selective about choosing the most appropriate strategies for solving the problem in question (Cardelle-Elawar, 1995). A variant of this approach was the 'reciprocal' teaching model advocated by Campione, Brown and Connell (1988) in which the teacher worked collaboratively with pupils to solve problems. In this model the teacher does most of the cognitive work at the outset

and decreases her or his contribution as pupils become more able to carry out the required procedures on their own. Yet another variant has been tried with college-level students by Schoenfeld (1987b), in which the teacher, by taking on a restricted role of scribe and orchestrator forces the students to focus on issues of self-regulation and control.

Problem solving in small groups. There are some generally accepted benefits of small cooperative group work. Pupils are encouraged, both explicitly and tacitly, to articulate their own thoughts and beliefs. Tacit encouragement comes from the lessening of the diffusion of responsibility for taking part which is likely in larger groups. They have opportunities to share the possibly differing approaches and starting points of their peers and as a consequence can, at various times, learn from and also teach, their peers. There are equally some disadvantages to group work, although it is harder to find these documented in the literature. The debate on the pros and cons of group work is more fully explored in Section 2.2.9

The teacher as a 'learner' role model for metacognitive behaviour. Some studies report on the results of work done in which the teacher makes a conscious attempt to be cast in the role of someone who does not know the answer and who must therefore demonstrate, in the search for a procedure which will lead to the solution, the desired kind of metacognitive behaviour. Schoenfeld (1987b) and Campione *et al.* (1988) both report some success with this approach. Lester, Garofalo and Kroll (1989a) on the other hand express some misgivings about attempts by the teacher to adopt such a role. Lester reported that he had encountered great difficulty in finding sufficient problems which would serve the dual purpose of challenging him at his own level and allow him honestly to take on the role of problem solver and at the same time be comprehensible and interesting to eleven year olds. He also soon fell back into the role of teacher and instead of modelling good problem solving behaviour he found himself falling back into an 'explaining' mode.

Videotapes. These have been used very successfully to develop pupils' awareness of their own thought processes and their ability to describe and regulate them. This can be done by showing pupils videotapes of others solving and discussing problems, but is more commonly used to show the subjects themselves performing problem solving tasks as a stimulus to help them recall and analyse their thoughts during the solution process. (Usnick and Brown, 1992; Peterson *et al.*, 1982; Schoenfeld, 1987b).

Most researchers who have tried to incorporate metacognition into their teaching have reported some degree of success in improving some aspects of their pupils' problem solving skills. Pramling (1988), for example, found that the groups of pre-school children she worked with who had been involved in 'metacognitive dialogues' showed a significant increase in their awareness of their own learning. Cardelle-Elawar (1992) noted that in an American study, those sixth grade students who performed well on complex cognitive tasks were those who possessed well-developed metacognitive skills. In a later study, in 1995, she also found that low-achieving pupils between grades three and eight who had received metacognitive "instruction", performed better in mathematical problem solving by improving in four aspects, namely by: (a) understanding how to approach a problem, (b) identifying the appropriate schema for organising the information, (c) recognising that there may be more than one way to solve a problem and (d) verifying their solutions. It is worth noting that the teachers involved in this study were all given three days training in metacognitive "instruction" and were briefed in the use of Mayer's (1987) model of problem solving "instruction", which is described in Section 2.2.5. Gardner (1991) remarked that the difference between being a good problem solver and a poor one, often lay in the ability to think about one's problem solving activities.

Lester, Garofalo and Kroll (1989b) found evidence in their work with eleven year-olds that (a) metacognitive processes develop concurrently with understanding of mathematical concepts, (b) that metacognitive instruction is most effective when it takes place in a domain specific context (that is, when problems were related to grade seven

content) and (c) metacognitive "instruction" is most effective when it is provided in a systematic, organised manner under the direction of a teacher.

These considerations have led to the formulation of a research question related to the development of children's metacognitive skills in the year-long problem solving programme organised for teachers by the researcher, which will be described in detail in Section 4.2.

### **2.2.7 Verbal reports as data**

As noted earlier in Section 2.2.5, which looked at the literature on frameworks for problem solving, one of the focuses of the present study will be pupils' reporting skills and the importance of embedding these skills within the framework chosen for this study. The work of researchers such as Schoenfeld and Garofalo and Lester, described in Section 2.2.5, has clearly established links between children's ability to report verbally on their problem solving processes and the growth of their metacognitive skills. Central to any discussion on children's acquisition of metacognitive skills is the question of how these skills are to be measured. It was decided, therefore, to include in the research questions one which sought to ascertain the extent to which children in year seven of primary school (P7), were aware of, could describe and could control certain of their cognitive processes. The proposed method of investigating these questions was by use of observation and verbal probes used by the researcher as the subjects were in the process of solving problems. The resulting responses would provide verbal data which would be transcribed for analysis of the resulting protocols.

In the last twenty years there has been some debate about the use of verbal reports as data although the discussion obviously goes back much further than this. In fact Brown (1987) traces the history of 'cognising about cognition' (p70) back to Plato and Aristotle. The more recent debate can be attributed at least in part to the seminal article by Nisbett and Wilson (1977) which proposed that subjects may have no direct introspective access

to some of their higher order cognitive processes. They held that subjects may be unaware of some aspect of the situation which may affect their response. They may be unaware of the existence of the response, or they may be unaware that their response has been affected by the stimulus. They also identified further limitations of verbal reports in that they might be fabricated or influenced by cues provided, however unwittingly, by the researcher. The expectations of the researcher could also influence their reports. Additionally reports may be incomplete and limited in their content due to the interviewee's lack of verbal ability and motivation. Nisbett and Wilson do concede that subjects may be able occasionally to report accurately on their cognitive processes. However it is possible for them to do this without having interrogated their memories about cognitive processes but by making what Nisbett and Wilson (p249) describe as, "a simple judgment of the extent to which input was a representative or plausible cause of output." In this sense their reports may be no more or less accurate than the predictions about their processes made by observers. Accurate reports by subjects therefore, do not necessarily imply direct introspective access to thought processes.

In a subsequent article which sought to refute many of these arguments, Smith and Miller (1978) argued that Nisbett and Wilson were implicitly imposing an impossible criterion for introspective awareness, namely that subjects should be aware of that which is systematically hidden from them by the design of the experiment. They asserted that failure to report accurately may not necessarily reflect a lack of introspective access but could be attributable to a variety of extraneous factors and influences such as apprehension about being evaluated, and the characteristics of the demands made on them. It is also possible that subjects may not understand the questions in the sense that the experimenter posed them and so may give unexpected answers, or that interviewers simply asked the wrong questions to elicit the hoped for type of response. Smith and Miller's conclusion was that the argument for the inaccessibility of mental processes was valid only in certain instances and that it was an overstatement to claim that such access was never possible. The correct question, according to Smith and Miller, was not whether introspective access to cognitive processes was possible or not, but under what conditions



such access would be possible. Two examples are quoted in their paper of cases when verbal self-reports seemed to have been accurate. These were the studies by Berl, Lewis and Morrison (1976) and Newell and Simon (1972) which Smith and Miller use as evidence to support a proposed dimension or criterion which will differentiate situations in which correct self reports will be possible from those in which they will not. This criterion is the extent to which the subjects are invited to report on tasks which are novel, interesting, relevant and engaging for them, as was the case in the two studies quoted, in which subjects were involved in choosing a college or solving challenging problems. This dimension of novelty and interest, the authors submit, was explicitly ignored by Nisbett and Wilson who admit (p242) to choosing experimental situations which were uninvolved to subjects.

The argument that verbal reports are, in certain situations, acceptable as data, was continued by Ericsson and Simon (1980) who used the theoretical framework of human information processing theory to propose a model to describe the verbalisation processes of subjects who were asked to think aloud or to give retrospective verbal reports of their cognitive processes. They identified different types of verbalisation which could describe specific events or could be of a more general nature. These were:

- talk or think aloud verbalisation in which all of the subjects' thinking is reported and the information reported is that which engages the subjects' attention at the time;
- concurrent probing and verbalisation in which subjects are invited to report on specific aspects of their work which are of interest to the observer;
- retrospective probing and verbalisation when subjects are encouraged to recall specific actions or events.

Their human information processing model held that (a) information recently acquired or attended to is kept in short term memory (STM) and is readily accessible for further processing (for example for producing verbal reports). (b) Information held in long term

memory (LTM) must first be retrieved (that is, transferred to STM) before it can be reported. (c) Concurrent verbal reports, which do not require the generation of new information, are reports based on information that is the focus of attention in STM. In response to the often expressed concern that the mere procedure of eliciting verbal reports can change the course and structure of cognitive processes, especially during concurrent verbalisation, they propose that when subjects articulate information that is directly available to them, this thinking aloud will not alter the nature of the cognitive processes. It might, however, slightly decrease the speed of the task performance. If, on the other hand, subjects are asked to produce information that was not normally available to them during the performance of the task and which had first to be decoded, then this could have an adverse effect on the cognitive process.

With respect to retrospective verbalisation, Ericsson and Simon predicted that subjects are likely to have direct access to their cognitive processes and information will still be held in STM provided the verbalisation instruction is given immediately after the process has been performed. When information must be retrieved from LTM, subjects' control processes can be more variable. However, the degree to which retrospective verbalisation must depend on retrieval from LTM can be minimised by keeping the interval between the event and the verbalisation request to a minimum, as mentioned above, and also by studying cognitive processes of short duration. All of these factors have implications for, and hence will inform the design of, the methods and procedures to be used in this study.

The debate about the use of verbal reports as data identified a number of accepted limitations in their use. A typical summary of these was provided by Goos and Galbraith (1996). The limitations which they identified were:

- reactivity - environmental influences (stress, researcher intervention and task demands) can affect cognitive processing.
- incompleteness - subjects may not report all the cognitive processes of interest to the experimenter.

- inconsistency - verbal reports may not correspond to observed behaviour.
- idiosyncrasy - generalisation is difficult because verbal reports are sensitive to individual differences.
- subjectivity - researcher bias influences the interpretation of the data.

They conclude that concurrent report methods with the instruction to report may provide the best description of cognitive processes during task performance since they do not require subjects to use inferential processes or to recall from long term memory.

There will be occasions when the conditions for providing direct introspective access to subjects' thought processes are not always present. At such times experimenters have needed to adopt measures to try to improve subjects' access to their LTM. A common approach in such circumstances is the use of a recall stimulus to aid retrospective verbalisation. The most commonly used such stimulus is videotape. There are several cases in which students were interviewed about their thought processes whilst viewing the videotape of the problem solving task they had performed, (Peterson, Swing, Braverman and Buss, 1982; Usnick and Brown, 1992; Randhawa, 1994; De Grave, Boshuizen and Schmidt, 1996). Another measure adopted was to act on the suggestions of Ericsson and Simon (1980) to overcome shortcomings of verbal reports referred to by Nisbett and Wilson (1977). Peterson and his colleagues did this by phrasing the interview prompts and questions in such a way as to provide the interviewees with no cues as to what the contents of their reports were expected to contain. They also assured the subjects that their responses would not be evaluated for correctness and finally they encouraged them to be as honest as they could in their reports.

Citing the cases of two individual students from the same study, Peterson and Swing (1982) observed that students' reported thought processes might well be better predictors of their achievement than would observations of their behaviour. In contrast to the views of Nisbett and Wilson they felt that students' verbal reports which indicated the extent

and nature of their understanding were informative and could usefully be applied to teaching.

The technique of 'stimulated recall' using an audiotape and a videotape was later tried and found to be successful by Usnick and Brown (1992). Their results showed that although gifted fifth grade pupils' minds were very active during a problem solving session, their thoughts did not always focus on strategies for solving the problem. Their minds tended to wander or they made judgements about the problem's difficulty, rather than thinking about the conditions of the problem or possible solution strategies for it. In general, however, the process which they referred to as 'Interpersonal Process Recall' stimulated by the viewing of the videotape, did succeed in eliciting responses from pupils which might not have been accessible using traditional assessment techniques.

In a later study, De Grave, Boshuizen and Schmidt (1996), noted that the recorded transcripts, or 'protocols', of verbal interactions showed only the tip of the iceberg of the phenomena they were researching and that the use of stimulated recall techniques provided more information than was available from the verbal reports alone.

Whilst most of the studies just mentioned related to retrospective verbalisation, there have been other examples of studies in which think aloud or concurrent verbalisation has been recorded, coded and used to provide information on subjects' problem solving processes. The study by Lawson and Rice (1987) claimed to show that concurrent verbalisation was possible and did provide insight into pupils' thought processes. The results were later discounted by Silver (1987) who, noting that the study had centred on one pupil only, described the results as "too meager" (sic) (p211).

Researchers using primary school aged pupils as the subjects of their studies have remarked in general terms on the difficulties of encouraging them to articulate their thoughts (Andrews, 1992; Logan, 1996). Lester (1987), commenting on the work done with his colleagues Garofalo and Kroll, noted that,

Asking problem solvers to 'think aloud', keep written records of their thinking, or work cooperatively with a partner have typically proved to be less successful than we had hoped. Thinking aloud during problem solving is often unnatural and sometimes has a debilitating effect on performance. Written retrospective accounts of one's thinking have provided very little information for us. (p265-6)

He goes on to suggest that this may be in part due to the children's lack of familiarity with this sort of activity or it may be attributable to their ages of between six and thirteen.

The study conducted by Desforges and Bristow (1995) adopted three strategies in an attempt to overcome some of these difficulties mentioned above. At the beginning of their study they asked the teachers from whose classes their subjects were being chosen to select only pupils who were articulate and who would enjoy participating. They then taught the pupils think-aloud techniques through demonstrations, explanations and practice. As the pupils engaged with the learning task and tried to give concurrent explanations of their thought processes, the interviewers attempted to maintain the pupils' verbalisation by the use of appropriate requests, questions and reminders. They concluded their work with the remark that although think-aloud and talk-aloud data are at best correlates of thinking in action they nonetheless constitute important data on the learners' thoughts.

The issues raised in the preceding discussion helped to inform the researcher in his choice of methods to be applied in using pupils' verbal reports as evidence of the level of metacognitive thinking being used by them during their problem solving. The pupil interviews, observations and interventions by the researcher, which resulted from consideration of these issues, will be described fully in Section 4.2.9.

### **2.2.8 The role of affect in problem solving**

An important part of the preliminary work of this study involved attracting participant teachers, and briefing them once they had agreed to become involved with the study. Part of this briefing was concerned with creating desirable classroom environments and

contexts within which problem solving could be nurtured. The researcher's own experience has long since led him to the view that pupils' own views, beliefs and feelings about mathematics can influence their performance in the subject. A desire to persuade participating teachers that this was also true for problem solving gave rise to a search of the literature which is summarised in this section.

In recent years there has been an increasing awareness of the importance of affect in mathematics education in general and in problem solving in particular. By 'affective domain' is meant the wide range of feelings, emotions, moods and beliefs which are regarded as being different from psychologists' typical model of cognition. In the USA the Commission on Standards for School Mathematics, (1987) reaffirmed the centrality of affective issues to mathematics learning. In England, *Mathematics Counts* (Cockcroft, 1982) and the *Guidelines* in Scotland also underlined the importance of positive attitudes towards mathematics and an awareness of the power and purposes of the subject. The relatively recent emphasis on problem solving has given new importance to the question of affect because of the recognition that when pupils work on problem solving tasks their affective responses tend to be more intense. This section will investigate evidence in the literature to support the assertion that pupils' performance in problem solving can be influenced by their own and their teachers' confidence, emotions, attitudes, and beliefs as well as knowledge.

A review of the literature up to the end of the 1980s revealed little research which had been done on affective issues in mathematics classrooms. The need to encourage more research in this field was commented on by McLeod (1989), Thompson and Thompson (1989), Cobb, Yackel and Wood (1989) and Silver (1985). Three general headings are commonly used to describe the affective domain in problem solving. These are, beliefs, attitudes and emotions. Each will now be discussed.

### 2.2.8.1 Beliefs

Under the heading of 'beliefs' will be included, pupil beliefs, teacher beliefs and societal beliefs.

Pupil beliefs. Many pupils believe that mathematics is about getting the right answer and being associated with certainty. These beliefs grow out of their experiences in school where 'doing' mathematics meant following the rules laid down by the teacher and 'knowing' mathematics meant remembering and applying the correct rule, (Lampert, 1990). Other pupils feel that mathematics is important, difficult and based on rules (Brown, *et al.*, 1988).

Perhaps the most comprehensive list of students' questionable beliefs about mathematics comes from Schoenfeld (1992) who has formulated the following views based on work done with university students over a number of years:

- Mathematics problems have one and only one right answer.
- There is only one correct way to solve any mathematical problem - usually the rule the teacher has most recently demonstrated to the class.
- Ordinary students cannot expect to understand mathematics; they expect to memorise it and apply what they have learned mechanically and without understanding.
- Mathematics is a solitary activity, done by individuals in isolation.
- Students who have understood the mathematics they have studied will be able to solve any assigned problem in five minutes or less.
- The mathematics learned in school has little or nothing to do with the real world.
- Formal proof is irrelevant to processes of discovery or invention. (p359)

It is important that teachers at both primary and secondary school stages should be aware of these kinds of beliefs and misconceptions held about mathematics by university students, since such awareness should help them to encourage and create a more realistic and positive set of beliefs amongst pupils at an earlier and more impressionable age.

Thompson and Thompson (1989), noting the absence of research-based guidance for teachers, offered some intuition-based recommendations for teachers which they hoped would lead to the formulation of specific "instructional" techniques. They referred to

Mandler's (1989) discrepancy theory which holds that when pupils' experiences do not match their expectations a strong emotional response is produced. If this is so, they argued, then it should be possible to trace children's affective reactions back to the beliefs and expectations from which they arose. An understanding of these beliefs and expectations will be necessary to deal with pupils' affect during problem solving instruction. This was a view also supported by McLeod (1989) who felt that if teachers wished all pupils to have a go at problem solving they would have to be prepared to understand and deal with the emotional stress which often accompanied initial attempts at problem solving.

In addition to having many preconceived notions about mathematics and problem solving, all pupils have beliefs about themselves. There are substantial gender differences in this area. Meyer and Fennema (1988) found that males tend to be more confident of their problem solving abilities, even when they have no cause to be, and the influence of confidence or self esteem seems to have a direct effect on pupils' abilities to solve problems.

Teachers' beliefs. There is a high level of agreement in the literature that teachers' beliefs will not only strongly influence the ways in which they present mathematics to their pupils, but also will determine the classroom atmosphere and the ethos in which mathematics is taught. Both of these factors, methods of presentation and classroom atmosphere and ethos, will, of course, result in implicit messages about the nature of mathematics as a subject being passed on to pupils. Thompson (1985, p286) quoted in Schoenfeld (1992) refers to two teachers whose quite different beliefs were reflected in their respective classrooms. One had a view of mathematics as a "finished product to be assimilated" and regarded her task as being "to disseminate information, and that of her students to receive it" (p360). The other saw mathematics as being about the discovery of ideas, mental processes, reasoning skills and solving problems. The atmosphere in her classroom was supportive of the development of her pupils' problem solving abilities. Thomson concluded that teachers' own beliefs about mathematics, passed on to



successive generations of their pupils, were themselves derived from teachers' own experiences as pupils. It is easy to see the importance of breaking out of what Schoenfeld referred to as "this vicious pedagogical/epistemological circle" (p360). One study in which this breakthrough occurred was referred to by Cobb, Yackel and Wood (1989) who set out to explore the emotional tone of a classroom in which the teacher had begun to encourage the children to verbalise their solution attempts. The children in the project classroom quickly learned that not knowing what to do was routine and the process of genuine problem solving became an overriding feature of their mathematics. The children came to understand the teacher's role as one of framing the problem solving situation and facilitating solutions and not providing answers. In short, they came to believe that mathematics was about doing problem solving. This change in beliefs created a much more positive problem solving environment in the classroom.

Societal beliefs. The current change in the approach to mathematics teaching taking place throughout the UK is at least partially attributable to the relatively poor performance of British pupils in international comparisons of mathematical achievement as measured by the *Third International Mathematics and Science Survey* (TIMSS, 1997; SOEID, 1996b). This survey noted significant differences in attainment not only between different countries, but also in some cases between different continents. The performance of children in countries such as Japan, Korea and Singapore, the so-called Pacific rim nations, easily surpassed that of pupils in the UK and North America and some commentators are attributing these differences to cultural or societal factors. In Pacific rim countries, success in mathematics is more commonly attributed to application and hard work than is the case in the UK. Pupils in Japan, Korea or Singapore who struggle to keep pace with their peers are expected to work harder for longer hours in order to keep abreast of the work of the class. Amongst teachers and parents, though not perhaps psychologists, in the UK and North America, there tends to be a greater belief in innate ability and a perceived consequent need on the part of the providers of education to cater for the particular needs of all the children in a class. It is hardly surprising that attempts

are already being made to change the way that the teaching of mathematics is viewed in the UK (SOEID, 1997; Department for Education and Employment, 1999).

It is not uncommon for parents to associate success in mathematics with adherence to 'traditional' approaches to teaching and learning (e.g. the importance of drill and practice on standard algorithms) and these attitudes and beliefs can be important factors in influencing the success of curricular reforms. Dillon (1993), quoted in McLeod (1994) reports on a case in which a new approach to teaching mathematics in one elementary school in the US was attempted but almost came to a halt because of the resistance of the local community to any type of approach which was seen as even mildly 'progressive'. Awareness of the importance of parental attitudes and beliefs has contributed to the growth in popularity of home/school partnership arrangements in many primary schools in the UK in recent years.

#### **2.2.8.2 Attitudes**

Attitudes towards mathematics tend to be reasonably constant and refer to pupils' feelings about the subject as a whole or about parts of it. They may, for example, think that work with shape is easy, that fractions are difficult, that numerical algorithms are boring and that problem solving is only for those who are 'good at maths'. Attitudes such as these can develop in two ways (McLeod, 1989). Firstly, they may become established as the result of a repeated and similar emotional reaction to the subject or topic. Whilst the emotional reaction may lessen in intensity over time, the attitude will be stable and will persist. A second source of attitude is the transfer of an already existing attitude to a new but related task. Pupils who have experienced difficulties with, for example, simple word problems in the early years of their schooling will be likely to expect to have similar difficulties with non-routine problems when these are first met further up in the school. The fact that attitudes, in common with beliefs, are relatively stable, does not however mean that they can not be changed. The challenge facing the teacher is to engender

positive attitudes by the way the subject is presented, by the way pupils are seen and valued, and by the choice of appropriate and interesting contexts and content.

### **2.2.8.3 Emotions**

Whereas beliefs and attitudes tend, in general, not to change quickly, some emotions are much less stable and are liable to more sudden change. A pupil who is depressed at his or her inability to solve a problem may become elated two minutes later when a solution is discovered. This instability of pupils' emotional reactions to mathematics may account for the lack of attention which has been paid to them as a focus for research. McLeod (1989) refers to a few researchers on cognition who have also noted the effects of emotions on cognitive processes in mathematics. Feelings of panic, fear, anxiety and embarrassment will all have negative effects on pupils' ability to think clearly about the task in hand. On the other hand pupils who confirm conjectures, make connections and solve problems will experience feelings of satisfaction and occasionally elation.

However under-represented the area of affect in problem solving has been, there is now sufficient evidence to suggest that there are positive measures which teachers can and should take to create positive beliefs, attitudes and emotions about problem solving and indeed that the affective domain must be taken more seriously when teachers are planning to teach problem solving. Some of these measures and techniques for creating positive contexts, in which pupils want to be involved, were explored with the teachers involved in this study during initial briefing sessions with the researcher and will be explained in the research methods in Chapter 4.

### **2.2.9 Collaborative group work in problem solving**

When this study was being designed, decisions had to be taken about the help which would be given to participating teachers from whose classes the pupil subjects would come. One such decision concerned the mode or modes of organisation of their pupils

which would best encourage problem solving behaviour and performance. Most primary teachers are familiar and comfortable with whole class, group, pair or individual working arrangements, all of which can be used profitably for learning mathematics at different times, depending on the content and context of the work being done. Those involved in this study, however, being for the most part new to problem solving, sought advice on the optimal organisation to use for problem solving. In particular they wanted to know whether small group, pairs or individual work was to be preferred. This section discusses the findings of the resulting review of the literature.

The strongest advocacy of the use of small groups in problem solving in the last twenty years has been provided by Vygotsky (1978) who hypothesised that a learner's potential for development at any time is limited to what he referred to as the 'zone of proximal development', or ZPD. When working alone, a young learner can function up to a certain level, but when working in partnership with other more capable children or perhaps with adult guidance, the learner may function at a somewhat higher level. This extra learning potential, which the learner will not achieve on his/her own, but may with some help, is the ZPD. Higher order skills are attained by first being reached collaboratively in the ZPD, then by subsequently being internalised on an individual level.

Children working in groups may be exposed to three or four possible avenues of investigation whereas, working alone, an individual might generate no more than one. Hence decisions have to be taken in the group situation and, before this can happen, any one individual must formulate one point of view, must listen to, understand and evaluate others and probably take part in a discussion about which is the best. Situations and events such as this will help to develop the self-regulation skills which are essential for problem solving (Schoenfeld, 1987b). In addition pupils will at times encounter disagreement and disbelief from their peers and will be forced to examine their own beliefs in order to be able to defend them. It is also likely that the group will collectively supply more background information and strategies than any individual. A beneficial spin-off of such group interaction is the concomitant development of spoken language

skills (Noddings, 1985). In cases where such interactive processes are being observed by an outside observer such as a teacher or researcher, the pressure on the learners is alleviated in small groups by what Goos and Galbraith (1996) refer to as "reassurance of mutual ignorance", (p234).

The authors sampled so far seem to suggest that small group arrangements are always beneficial. There are, however, some who entertain some reservations about their use. Forman (1989), for example, in her study of the benefits of peer group collaboration, specified three conditions which she felt must be fulfilled before a ZPD created by collaborating subjects could be effective. These were:

1. pupils must have a mutual respect for each other's perspective
2. there must be an equal distribution of knowledge
3. there must be an equal distribution of power.

A breakdown of any one of these conditions could lead to failure of the collaborative process and hence teachers had to exercise some care in the composition of their pairings or groupings. However, it seems likely that condition 1 is the most important of these, given that ZPD can be effective between parents (and teachers) and children who are unequal in respect of knowledge and power.

Forman also further explored the notion of learning within the ZPD, by referring to the idea of 'proleptic instruction' which had been introduced by Stone (1985) and subsequently discussed by Campione, Brown and Connel (1988). Proleptic "instruction" characterised a type of learning and teaching within the ZPD in which the learner is required to understand the presuppositions of the speaker. Further,

since the speaker's presuppositions are left unstated, the listener must construct them for himself in order to understand fully the intended message. This process of construction makes the message more alive for the listeners (Stone, 1985, p135)

This notion of proleptic "instruction" would seem to apply not only to adult-child "instruction" but also to peer "instruction" and hence has a relevance for the

consideration of small group work in problem solving. There are two significant differences between adult-child and peer proleptic "instruction" in their degree of what Forman called their 'complementarity' and their 'reciprocity'. In an adult-child "instructional" situation the adult typically has more power and knowledge than the child, so one teaches whilst the other learns. This is what is meant by complementarity of interaction. In peer instruction, however, power and knowledge are more likely to be more equally shared, each child taking on the roles of learner and teacher at different times. This is referred to as reciprocity of interaction.

A second difference between the two types of proleptic situations is that children in a cooperative peer work group can learn to share the responsibility for task setting and strategy selection, whereas in an adult-child situation it is more likely to be the adult who assumes the role of goal setting and task definition.

Forman concluded that proleptic "instruction" can play an important role in peer work groups. In her study the two girls who were the subjects of it were able to create a bi-directional zone of proximal development by assuming the roles of teacher and learner at different times. Since this type of learning is mostly implicit its effects needed to be measured over a prolonged period of time. Implying that she was less than totally convinced of the benefits of small collaborative group work, she noted that more research would be needed to identify factors which maximise the cognitive benefits of peer collaboration. Perhaps a degree of caution should be exercised with respect to this piece of research which based its conclusions largely on work done with only two seventh grade girls.

Other researchers too are not wholly committed to the use of small groups. Buchanan, (1987) and Cobb, Yackel and Wood (1992) agreed with Forman that more research was needed to describe and evaluate the complex interactions which took place in such situations. Noddings (1985) who initially professed agreement with Vygotsky, later became less sure of the advantages of small group work when her own study seemed to

show that thoughtful whole class instruction could produce as good or even better results than small groups - a sentiment that would find favour with the authors of the *National Numeracy Strategy* (DfEE, 1999) in England and *Improving Mathematics Education 5-14* (SOEID, 1997) in Scotland.

Lester (1987) and Lester, Garofalo and Kroll (1989a) also harbour reservations about small group work. They noted that the teacher relinquishes a degree of control over the "instructional" activity when children are working in small groups. A lot of the teacher's attention is removed from teaching mathematics and is devoted instead to classroom management matters. This is another argument which would find favour with the promoters of the *National Numeracy Strategy* (DfEE, 1999). Referring to his work with 6-13 year-olds, Lester noted that,

Cooperative work in small groups has been cited as a natural way to get students to talk aloud and to share their ideas openly. Unfortunately our experience has been that most students find it quite difficult to do this. (p266)

This sentiment is repeated in the 1989 study by Lester, Garofalo and Kroll who note that,

we simply know far too little about the conditions under which cooperative learning groups truly enhance student problem solving. (p91)

On balance there appears to be no consensus on whether small group work in problem solving is to be recommended or not. It may be worth noting that those researchers who tended to come down in favour of it, conducted studies primarily but not exclusively with older school pupils or college students, whilst those expressing reservations had worked primarily with younger children. It may be reasonable to surmise that there are more benefits to be derived from small group work with older learners than with younger ones and this conjecture will be used to inform the advice given to the teachers in this study.

### 2.2.10 Assessment and protocol analysis

This section will consider some of the literature on assessing problem solving and in particular will look at attempts which have been made to analyse pupils' or students' problem solving 'protocols'. This use of the word 'protocol' seems to have developed in American research circles and is now used on both sides of the Atlantic (Whitebread, 1996), commonly to refer to, "transcripts of recordings made by people solving problems 'out loud'" (Schoenfeld 1985b, p183). Different protocol coding schemes have been used to attempt to provide what Schoenfeld (1983, p347) refers to as "an objective record or traces of a sequence of overt actions taken by individuals in the process of solving problems".

This study will not only focus on pupils' ability to get the correct answer to problems but also on the role that metacognition plays in the process and whether this affects their awareness of problem solving strategies. Consideration of these factors suggests that it is appropriate in this section to look not only at formal or summative assessment techniques but more importantly at informal and formative techniques which will cast light on pupils' developing strategy awareness and other related metacognitive skills. To achieve these ends the literature strongly indicates the use of protocol analysis to support other assessment techniques.

A first reading of the article by Silver and Kilpatrick (1988) would suggest a rather bleak picture in the area of assessing problem solving. They note, commenting on the use of standardised tests in the USA, that many have a sub-section on problem solving but that they demand nothing more than the simple routine applications of a few well-worn algorithms. Remarking that the 'ultimate instrument' (p185) for assessing problem solving is the teacher, they conclude that much more research is needed in this area and it is towards the teacher that it should be directed. They do refer to two sources of hope for the future of problem solving assessment. These are to be found in the work of Marshall (1988) and Goldin (1982).



In his article, Goldin discusses the difficulties of testing problem solving and suggests that more light can be shed on strategy usage by presenting subjects with structured questions to be asked by the teacher, either during or after the problem solving episode. He repeats the warning that questioning to encourage concurrent verbalisation (though he does not use these terms which were not in currency in 1982) may affect the problem solving outcome. He quotes Dienes and Jeeves (1965, 1970) as having found a positive relationship between retrospective accounts of strategies used and observed behavioural strategies, unlike Branca and Kilpatrick (1972) who found that retrospective evaluations frequently did not correspond to their measured strategy scores. Both types of results were subsequently confirmed in the work of Ericsson and Simon (1980), previously referred to in Section 2.2.7. Goldin (1982) noted that structured interviews can be useful but, to ensure that response patterns can be meaningfully interpreted, categories of responses must be established in advance.

Marshall (1988) mentions three things that teachers should have ascertained before assessing problem solving. Firstly they must know whether the learner has enough factual knowledge to be operating within the specified domain. Secondly they need to know that the learner has the required behavioural alternatives or tools and finally they need to know whether the learner can call upon the knowledge and skills in a non-predetermined way to make sense of the new experience. The first of these can be taken care of by ensuring that all calculations or algorithms required to solve a problem are well within the scope of the learner, otherwise the assessment becomes one of arithmetical techniques as opposed to problem solving. The study which is the subject of this thesis will focus on pupils' ability to achieve the second and third of these three aspects, which to an extent will depend on factors such as experience of solving problems successfully, awareness of a range of strategies and the confidence and self-belief that come from both of these.

Work done by Artzt and Armour-Thomas (1990) had some relevance to the present study in that their work was conducted with twenty-seven seventh grade pupils. Their starting point was that recent studies had suggested that difficulties encountered by many pupils in problem solving lay in their inability actively to monitor and subsequently to regulate their cognitive processes, so they decided to investigate the heuristic and cognitive processes that occur when seventh grade pupils work in small groups. One of the products of their study was a framework for protocol analysis, which was previously described in Section 2.2.5. This was an adaptation of that devised and used by Schoenfeld (1987b), which used a structure of episodes and executive decision points, described more fully later in this section. Their pupils were divided into six groups and they each spent fifteen to twenty minutes on a problem. The behaviour of each group was analysed and categorised into one of 'cognitive', 'metacognitive' or 'watch and listen' classes of activity. The group which spent the highest proportion of its time on cognitive activities and the least proportion on metacognitive thinking was the only group not to solve the problem, leading the researchers to conclude that they had been caught up in the doing of the problem rather than in thinking about what they were doing. The results which were produced by an analysis of these protocols suggested the importance of metacognitive processes in small group problem solving, but they should be treated with caution as they were derived from only one class doing one problem. The results did, however, suggest the need to attempt to categorise responses according to some predetermined classification.

Another significant piece of work was done in Portugal by Fontana and Fernandes (1994). Their study discusses earlier work by, for example, Boud (1989) and refers to the benefits accruing to students in higher education from the use of self-assessment. The authors wanted to try similar techniques with primary age children since they felt that the use of self-assessment techniques had not been explored with this age group. It had been argued that primary pupils' cognitive immaturity would prevent them from carrying out such a process with acceptable accuracy (Nicholls and Miller, 1983). The researcher has been unable to find any references to attempts other than his own in the present

study, by young (primary age) children to assess or describe their own use of problem solving strategies in a systematic way. Quoting from the literature, (Boud, 1990, Daines, 1985, Descombes and Robins, 1980 and Lublin 1980) Fontana and Fernandes assert (p415) that,

Frequent self-assessment produces a greater impact upon children's thinking and behaviour, carries an important practice effect, may influence pupils towards a better organisation of the previous learning upon which future learning is based and may serve to sharpen their perceptions of the objectives to be achieved.

This appears to support the decision taken in this study, which grew out of the literature reviewed in Sections 2.2.5 - 2.2.7 and which is described more fully in Chapter 4, to encourage one group of classes in the study to adopt a practice of frequent self-reporting of their problem solving processes and strategies. Although it may be argued that such use of reporting strategies requires a deeper and more metacognitive set of processes than is required for assessing their own work, it is the researcher's belief that the former would be likely to share some, if not all, of the same benefits. It is hoped that the present study will be able to explore whether this is in fact the case.

The final protocol analysis scheme described here is the one which has been perhaps the most influential of all those mentioned in the sense that it has spawned a number of spin-off versions used by other researchers. This is the one devised by Schoenfeld (1987b). In an earlier article in 1985 he had argued two reasons for using what he referred to as 'pair protocols', effectively protocols or records of problem solving processes produced by students working in pairs. He argued that two students would provide more verbal data than students working as individuals as they must fully justify and explain their proposed actions to each other. Also the shared insecurity of working together lessens some of the pressure of working under observation, which of course is necessary if a protocol is to be produced. Since they may well start from different points, they will need to discuss, reflect on and monitor their own and each other's thinking, as well as making this activity observable by the experimenter. This model, with modifications suggested by

the work of Mason, Burton and Stacey (1985) was subsequently used by Goos and Galbraith (1996).

In his 1987b and 1992 articles, Schoenfeld described his framework for analysing protocols of college students' problem solving work. His framework has the six elements, Read, Analyse, Explore, Plan, Implement and Verify and he developed a time-line graph to illustrate how different students approach a particular problem. These graphs show clear differences between 'novice' and 'expert' behaviour. The novice will typically spend a short time reading the question before attempting a solution. This attempt may last for a prolonged period of time and still get nowhere. An expert solver on the other hand will spend time on all elements of this framework, revisiting some and constantly asking himself where he is and where he is going. These two types of behaviour are shown on Schoenfeld's graphs of a novice and expert problem solver from his 1987(b) paper, reproduced here as Figures 2.1 and 2.2.

Figure 2.1  
*Schoenfeld's graph of typical 'novice' problem solving behaviour*

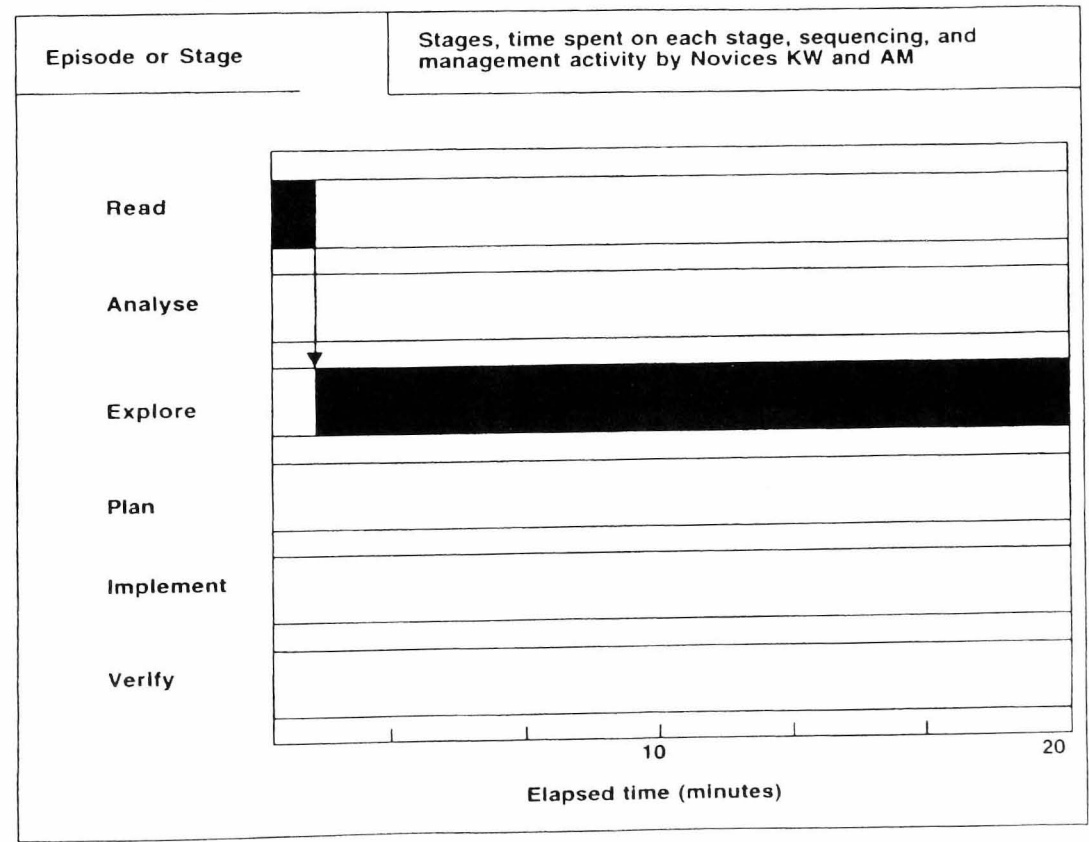
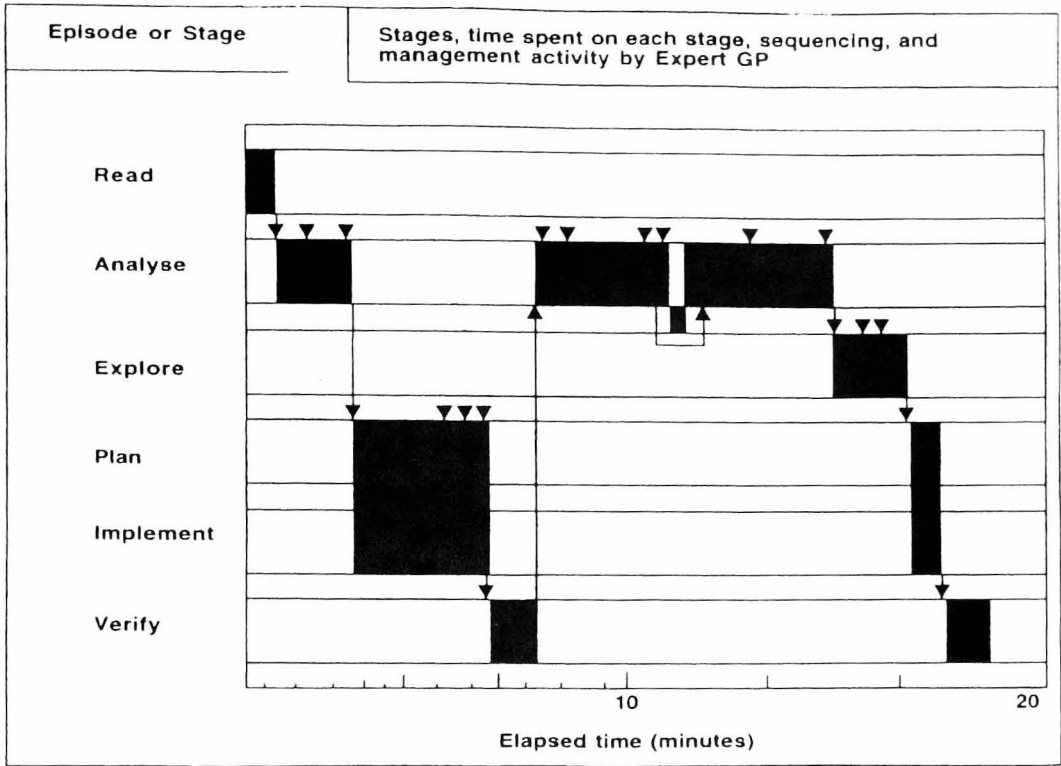


Figure 2.2  
*Schoenfeld's graph of typical 'expert' problem solving behaviour*



The black rectangles in these graphs represent what Schoenfeld referred to as 'episodes' which he defined as "macroscopic chunks of consistent behaviour" (1983, p354). At the beginning and end of most of these episodes and at points during some of them, represented on the graphs by small inverted triangles, were points at which managerial or executive decisions took place. It was at these that expert subjects took strategic decisions of a metacognitive nature to inform themselves of the next direction their problem solving activity should take.

In a later paper, Schoenfeld (1992) described the protocol parsing scheme illustrated by these figures as good only for certain types of 'non-interventive' problem solving. It was useful in documenting the presence or absence of executive decisions and demonstrating the consequences of these decisions. Such control behaviour, he noted, was unlikely to be needed in the performance of routine or algorithmic exercises and moreover the method reveals little or nothing about the mechanisms underlying monitoring or assessment. More interventive methods would be needed to probe reasons why individual subjects did, or did not, follow certain options and these probes might themselves interrupt the flow of the problem solutions. The scheme, it should be noted, was designed to be used

by college level students who were presumably reasonably able to describe their processes, to allow such protocols to be composed.

The framework which was chosen for use in the study reported here was the Starting - Doing - Reporting one described in the *Guidelines*. There were two reasons for this. Firstly it was a simpler framework than Schoenfeld's and others mentioned above, which meant that it was easier to administer by one researcher working as an individual. Secondly, it was one with which teachers would have some familiarity and, thirdly, it was one which, the researcher hoped, would allow for interventions and probes to try to monitor pupils' levels of metacognitive activity. As explained more fully in Section 4.2.9, Schoenfeld's two key ideas of 'episodes' and 'executive decision points' were found useful in deciding on the structure of pupil interviews and the subsequent analysis of the resulting protocols.

### **2.2.11 Conclusions**

This concluding section of the chapter will describe how the literature has informed decisions about the design and method used in the study of children's acquisition of problem solving skills.

The meaning of the word 'problem' used in the rest of this thesis will firstly be clarified. The researcher has chosen the designation of "process problems", suggested by Charles and Lester (1984b) as the most appropriate and succinct description of the problems used with the pupils involved in the research. These were described earlier, in Section 2.2.2, as

self-standing, non-standard problems which have one identifiable solution, which can be solved by any one of a number of different processes and in which the process itself as well as the solution can be of interest. (p36)

The third category of problem identified by Goldin (1982) also provides an apt description of those used in this study. Problems in this category were described as those

for which the subject does not know the answer but does possess a procedure for getting it. The subject cannot, however, state with certainty that he or she possesses the procedure until after the problem has been solved.

These descriptions and the problems which match them were chosen as they best approximate those illustrated in the *Guidelines*. The latter were described, rather vaguely in the researcher's opinion, as "tasks designed specifically to highlight certain approaches to mathematical thinking" (SOED, 1991, p48). Because these were exemplified in the *Guidelines*, these were the types of problems which teachers in Scottish primary schools were trying to use in their attempts to implement them.

Section 2.2.4 referred to the importance placed by most researchers on problem solving strategies. The lack of agreement on how pupils were to acquire a knowledge of the most common of these strategies together with an ability to use and apply them appropriately in new situations, suggested a need for further investigation. This lack of consensus on how strategies should best be taught and learned, led to the formulation of a research question (Chapter 3), which in turn gave rise to an investigation which became one of the main focuses of the study reported here.

At the time the study was begun, many teachers, and virtually all of those involved in the study, were new to the teaching of problem solving. All they had been given, in terms of support material, was a copy of the *Guidelines* which contained two sections on problem solving and enquiry. In these circumstances it was obviously sensible for teachers to work within the framework provided in the *Guidelines* and which was described in Section 2.2.5. Although a number of other frameworks were considered and discussed in that section, the fact that one had been provided for Scottish teachers and that it contained an unusual 'reporting' element (Table 2.1), persuaded the researcher of the need to retain it as the framework to be used for the background to the work to be done in schools.

Some researchers have commented on how developing children's general metacognitive skills can improve some aspects of their mathematical performance, (Pramling, 1988; Cardelle-Elawar, 1992; Pressley, 1994; Carr and Biddlecomb, 1998; Hacker, 1998). Despite this acknowledgement of the desirability of encouraging metacognitive growth, as discussed at some length in Section 2.2.6, there is as yet no agreement on how these metacognitive skills are best taught and learned. In recent years Davidson and Sternberg (1998) and Carr and Biddlecomb (1998) have remarked that a great deal still remains to be discovered about the role of metacognition in problem solving and that little is known yet about how children acquire metacognitive knowledge. Given this acceptance of a continuing need for more study, the researcher concluded that the existence of the 'reporting' element in the Scottish problem solving framework might provide a context in which metacognitive skills could be developed and that the need to 'report' might also link metacognitive growth to a greater awareness of and confidence in the use of problem solving strategies. These considerations led to the formulation of a research question about the link between 'reporting', metacognitive growth and knowledge of problem solving strategies. The mechanisms for creating a context in which this question could be studied, will be explained in Chapter 4, Section 4.2.7.

The difficulties of gaining access to children's metacognitive processes have been well documented and have been discussed in Section 2.2.7. The literature has supported the researcher's belief that such access is possible provided the pitfalls and difficulties are recognised and measures taken to minimise them. The work of Smith and Miller (1978) and later of Ericsson and Simon (1980) has been used to inform the design of the interviews used with pupils in this study and has suggested the use of concurrent and retrospective reports, the latter given immediately after the problem solving process has taken place. Results from other sources, referred to in Section 2.2.7 on verbal reports, have been taken into account in the design of the pupil interviews and the content of the problem solving exercises used, both of which are further described in Chapter 4.



At the outset of this study the researcher had to acknowledge the impossibility of exercising control over most aspects of the behaviour of the fourteen teachers involved, during their work on problem solving. Those few aspects over which control was attempted are explained in Chapter 4. Although the teachers' own attitudes to mathematics and the classroom environments they provided were considered to be, to a great extent, beyond the researcher's control, attempts were made, during briefing meetings with the teachers, to suggest actions which could be taken to create positive attitudes and build confidence amongst the pupils in the study. The importance of trying to do this has been discussed in Section 2.2.8. and to this end teachers were provided by the researcher with a set of materials which would support their problem solving activities for a whole school year. The materials were categorised into five strategies and three levels of difficulty, with advice on how they could be organised and used by the teachers in a way which would provide pupils with opportunities to experience success whilst still being challenged by stimulating and interesting problems. More details of these materials are given in Section 4.2.5.

The literature related to the use of collaborative group work in problem solving, discussed in Section 2.2.9, was inconclusive, although there did seem to be some agreement that small group work was more successful with older pupils than with younger. Because of the lack of definitive evidence supporting any one particular organisational mode for problem solving, it was decided to discuss the various possibilities with the participating teachers and to take account of their own habits and preferences before making any explicit recommendations. In the briefing meetings held with them, the consensus was that they would prefer their pupils to try the problems on their own initially, before looking for mutual support and discussion within small groups varying in size from two to four. Subsequent discussions with the teachers suggested that this is in fact what happened. Most of the teachers remarked that their pupils seemed to want to experience the satisfaction of arriving at their own solution before discussing with their peers the answers found and the processes used. None of these arrangements seemed to be at odds with the findings of Section 2.2.9.

As explained in Section 2.2.10 above, some of the ideas found in the literature proved to be useful in the design and structure of the interviews held with the pupils while they were actively engaged in problem solving and in the subsequent analysis of the protocols produced by recording the interviews on audiotape. Other aspects of the assessment of pupils' problem solving performance owe little or nothing to the literature as very little was found relating to the assessment of problems similar to those used in this study with children of the same age. The pre-tests and post-tests used were designed by the researcher and other assessment data were collected by interviews with the teachers. Further details of assessment and protocol analysis are given in Chapter 4.

In conclusion, the study of the literature related to curriculum evaluation has enabled the researcher to identify, from those historical research paradigms discussed, elements of the classical, illuminative, democratic and responsive models of evaluation. The inclusive nature of this combination of various models allowed the researcher to take account of, and respond to the views and concerns of teachers and to use their responses to the national survey to select and develop the second major part of this study. These views have confirmed the researcher's own belief that a situation existed in many Scottish schools which most teachers were not trained or equipped to resolve without help, and which this study is intended to address. In this sense the research which is the focus of this study can be described as 'practical'. Nisbet (1999) wrote that:

Educational research can be classified under four heads: philosophical, historical, empirical, practical. The fourth of these is in the context of specific educational settings, tackling practical issues which arise in specific situations and which have to be solved (and understood) in that setting. This is the kind of research which is increasingly being done in education, and the academic world is slow to understand and accept it as a contribution to 'knowledge'.

The study of the literature related to problem solving has highlighted issues and concerns and the national survey identified specific situations related to the teaching of problem solving which exist in many primary classrooms. The research questions which have been formulated in an attempt to help to resolve these particular situations are articulated in the following chapter.

### 3.1            Introduction

The research questions articulated in this chapter fall into two categories relating to the two component parts of the study. The first two refer to the evaluation of the mathematics *Guidelines* and the remainder to the in-depth problem solving study.

As noted earlier in Section 2.1.2, the researcher's primary aim was to evaluate the *Guidelines* through the eyes of the practitioners who have responsibility for implementing the changes in schools. It is also anticipated that, from the responses given by teachers, important lessons for future curricular change might be learned. Research questions 1 and 2 grew out of these considerations.

The researcher's own experience and interest in problem solving in primary mathematics were explained in Chapter 1 and in Chapter 2 the literature relating to some aspects of problem solving was reviewed. Questions 3 to 6 will address a number of the issues referred to in Chapter 2.

### 3.2            Research questions

#### Question 1.

*How do primary teachers view the contents and implementation of the 5-14 National Guidelines in Mathematics?*

This question was intended to elicit all of the major issues faced by teachers in their work with the *Guidelines*. Comments should be made about the style and the content of the *Guidelines* and teachers should identify the main changes resulting from them. Support

mechanisms made available to teachers to help them with implementation should also be commented on and the main difficulties encountered by teachers should be specified.

### Question 2

*Which of the views expressed by teachers in this survey should inform future curriculum innovations?*

It is expected that there will be a number of issues on which teachers will be in general agreement and which will be of interest to those responsible for future curricular changes. This should be the case whether the issues identified are positive or negative in nature.

### Question 3

*What were the effects of a year-long structured programme of problem solving in mathematics as perceived by (a) the teachers and (b) the pupils, of the P7 classes involved with it?*

This question was posed to determine whether teachers and pupils have similar views on the effects of the programme. It is hoped that teachers will be able to identify positive effects on both their own and their pupils' practice and attitudes. The effects of the programme on pupils, in both the cognitive and affective domains which this question will seek to identify, might also contribute to the information being sought through questions 4, 5 and 6.

### Question 4

*To what extent does frequent practice in 'reporting' affect pupils' (a) metacognitive abilities, (b) knowledge of strategies, (c) problem solving skills?*

The 'reporting' component of the problem solving framework advocated in the *Guidelines* is unusual and is seen by the researcher as a potentially effective factor in developing

pupils' skills and understanding in the three areas mentioned in the question. The question is designed to test the research hypothesis that encouraging pupils to describe and articulate the methods, processes and strategies used (that is to 'report'), will have positive effects in these three areas.

#### Question 5

*To what extent can the 'reporting' aspect of the problem solving cycle as defined in the Guidelines be justified as a component of the problem solving framework?*

To some extent the answer to this question will depend on the results of question 4. There may, however, be benefits other than those specified in question 4 which will justify the inclusion of reporting in the problem solving framework.

#### Question 6

*What effects has the year's structured programme had on pupils' and teachers' beliefs, views and attitudes to problem solving?*

This question relates to some of the discussion in Section 2.2.7 about the role of affect in problem solving. The inferences drawn from the literature suggest that any improvements in the affective domain will have corresponding benefits in the cognitive domain.

Table 4.1 in Section 4.2.13 of the following chapter shows the links between the research questions and the data collection methods used.

## Chapter 4 Research Methods

### 4.1 The National Survey

#### 4.1.1 Introduction

This study of the implementation of the *5-14 National Guidelines in Mathematics* in Scottish Primary Schools was carried out between January 1995 and May 1997, with the support of a grant from the Scottish Office Education and Industry Department. Although similar studies had been done of the implementation of National Curriculum Mathematics in primary schools in England and Wales, (Brown *et al.*, 1993; Bennett *et al.*, 1992), no such study, devoted exclusively to primary mathematics, had been conducted in Scotland.

When designing the study, the researcher had to take into account the variety of historical paradigms which had informed curriculum evaluations in the past. As a result, the study incorporated elements of a number of different research paradigms, all of which were discussed in Chapter 2.

The national guidelines for the education of pupils in the 5-14 age range in Scotland had their origins in a paper published in 1987 by the Secretary of State for Scotland, *Curriculum and Assessment in Scotland: A Policy for the 90s*. This paper was cited in the *Guidelines* (SOED, 1991, p.vi) as having identified a need for:

clear guidance on what pupils should be learning in primary schools and in the first two years of secondary schools;

improved assessment of pupils' progress

better information for parents about the curriculum and about their children's performance.

However, when the guidelines in each different subject were written, no clearly defined set of objectives was specified for them. Each Review and Development Group was given the remit of undertaking a "wide-ranging review of good practice" in order to:

set out clearly the knowledge, understanding, skills and attitudes appropriate to its curricular area; and then to advise on the formulation of national curriculum guidelines... (SOED 1991, p.vi).

The absence of any specific set of curricular objectives precluded the exclusive use of a classical model of evaluation, suggesting instead the flexible use of more recently developed paradigms. These focused not only on expected outcomes, however underspecified these were, but also on unintended outcomes which take account, in terms used by Parlett and Hamilton (1976), of both the "instructional system" and the "learning milieu". Hence there were, in the study, elements of Scriven's (1973) Goal Free Evaluation model which he proposed was useful, if not necessary, when no objectives were specified. Similarly aspects of Stake's (1975) "responsive" evaluation were included in the study since it sought to provide as much useful information as possible to people who were directly or indirectly involved with the curriculum. By seeking to provide information about the innovation to a wide audience, without any restriction placed on the evaluator, the study can also claim to fit the description of MacDonald's (1976) "democratic" model of evaluation. In addition it continues a trend, begun in the 1980s and continued up to the present time, to involve teachers in the process of evaluation.

Whilst these comments suggest that there were many qualitative aspects to this piece of work, the researcher was nonetheless aware of the need to heed the advice of, for example, Parsons (1976), who stressed the continuing need for the newer more qualitative paradigms to retain a degree of the "sophistication and rigour" of previous models and Stenhouse (1979) who advocated a retention of quantitative ingredients in evaluation research to help maintain "standards". As a consequence of these considerations, this study will include aspects of both qualitative and quantitative methods.

### 4.1.2 The pilot study

Before the pilot study took place, a pre-pilot exercise had been conducted in November 1994. At this time seven practising primary teachers were asked to complete a questionnaire consisting of ten sections and one hundred and eighteen individual items. They were also invited to respond to some questions about the questionnaire with a view to identifying any shortcomings in it. In particular, each teacher was asked to comment on its length, the clarity of the questions and the appropriateness of the issues being addressed, as well as to suggest any additional sections. All seven teachers were subsequently interviewed by the researcher.

In light of the comments received from the respondents to this questionnaire, some minor changes were made to it. The amended version was then used in the pilot study.

The pilot study, conducted in Fife, Grampian and Tayside Regions, involved 40 primary teachers, plus one adviser and two staff tutors. The schools and teachers contacted by the researcher had been suggested by the regional primary advisers as those who were thought to be most likely to produce a response to requests for assistance in data collection. This means of identification of participating schools meant, however, that any results obtained could not be considered to be representative of the views of the teaching body as a whole. Given that two of the primary purposes of the pilot study were to test the data collection instrument and identify issues for further investigation, the non-random nature of participant selection was not considered to be problematic.

Each of the participating teachers completed a questionnaire and 15 of them were also interviewed.

The pilot study had four main purposes, all of which would help to give focus to the national survey. The first was to test the data collection instrument which the researcher was proposing to use in the national survey. The second and third were to identify those



aspects of the *Guidelines* which had resulted in significant changes in primary mathematics teaching, and which had caused concern or difficulty to teachers. The fourth purpose was to identify an area or areas for a more in-depth research study to be carried out after the national survey. The area so identified will be discussed fully in Section 4.2.

#### **4.1.3 Results of the pilot study**

As a result of the pilot study, seven areas were chosen for investigation in the national survey. These were:

1. teachers' views of the *Guidelines*;
2. effects of the *Guidelines* on classroom practice;
3. support available to help implementation;
4. teachers' views on the Problem Solving and Enquiry outcome;
5. the specific issue of calculating;
6. the use of microcomputers in primary mathematics;
7. the use of context in mathematics.

Of these areas, the researcher felt that 1 and 2 would be of interest to the SOEID, local authorities and other curriculum developers in providing evidence of the broad general effects of the *Guidelines* and teachers' reactions to them. The findings for area 3 would, it was hoped, contain information about those strategies and agencies which teachers found were most helpful at the implementation stage of the *Guidelines*, as well as identifying other supportive measures which they would have liked to have had provided. If this were to be the case, the findings might be of use in future curriculum developments. Categories 4 to 7 were included as those parts of the contents of the *Guidelines* which the pilot study suggested were providing teachers with most challenge.

#### 4.1.4 The national survey

Preparations for the national survey began in September 1996 when letters were sent to the Directors of Education in all 32 regions of Scotland, explaining the purpose and nature of the survey and asking for permission to approach the headteachers of a randomly selected number of schools from their regions to invite some teachers in their schools to take part in the survey. Every local authority approached in this way gave its approval for schools to be approached, although some expressed misgivings about potential overload on some schools which might have been involved in other research projects or other whole-school activities such as school inspection processes. In an effort to avoid approaching schools on which other recent research or similar demands might have been made, the researcher was able to acquire information on the schools which had been used for two other major national surveys in recent years. One such survey was carried out on behalf of the Scottish Office Education Department's Committee on Testing by the 5-14 Assessment Unit of the Scottish Examination Board in 1994 and 1996, and another was done simultaneously by the Scottish Council for Research in Education (SCRE), in conjunction with Northern College of Education. Both of these bodies were good enough to supply the researcher with the lists of schools approached in the course of their surveys. The researcher was then able to delete the names of these schools from a national database of Scottish schools acquired from SCRE. In addition, any schools which Directors of Education had referred to in their correspondence with the researcher as having been involved in other projects were also deleted from the database, before a random selection was done from those schools remaining. A disadvantage of these procedures was that the selection eventually made was not drawn from every primary school in Scotland. This, however, was compensated for, in the researcher's view, by the fact that there would be a better chance of obtaining a higher response rate from schools, the majority of which had not recently been involved in other surveys.

According to the *Scottish Education Statistics* of 1995 (SOED, 1995), the latest set of figures available at the time of the survey, there were 2341 state primary schools in the country, employing 22500 teachers. Selecting every sixth school from those remaining on the database yielded a sample of 390 schools from all 32 regions of Scotland.

The survey was conducted in late 1996 by means of a 39-item questionnaire (Appendix 4.1) which was sent to 913 primary teachers in the 390 selected schools. Each of the schools was sent either 1, 2 or 3 questionnaires depending on its roll. A letter to the head teacher asked for the questionnaire to be given to any one teacher in schools with fewer than 70 pupils. In schools with between 71 and 150 pupils, head teachers were asked to invite responses from two teachers, one from the early stages and one from the upper stages. For schools with rolls in excess of 150 pupils, responses were sought from three teachers, one from each of the P1-3, P4/5 and P6/7 stages of the school.

Whilst the method of sampling the schools to be used in the survey fits Cohen and Manion's (1994) description of 'systematic sampling', the decision to invite responses from teachers at designated stages within the schools falls within their designation of 'stratified sampling'. This decision grew out of a desire by the researcher to make the sample more representative of all Scottish primary teachers. It is, however, acknowledged that by doing this the randomness of the sample has been diminished.

Of the 913 questionnaires distributed, 328 were returned. This 36% return rate was gratifying to the researcher, given the pressure of work in schools at the time and the demands being made on primary teachers, who were still coming to terms with the implementation of the national guidelines in all curricular areas.

The results cannot claim to be representative of the views of Scottish primary teachers as a whole, because of a number of variables over which the researcher had no control. There was no way of ascertaining, for example, how teachers in any school were invited or selected to complete the questionnaire, or indeed whether some may have volunteered.

It is possible that some may have volunteered or been selected because of a special interest in primary mathematics or because of their strongly held views about the *Guidelines*. The researcher could only hope that teachers who responded represented a good balance of teachers of the different stages of primary schools and of those who had a range of views about the *Guidelines*. The researcher had considered extending the survey to include headteachers and pupils in an attempt to introduce an element of triangulation. To replicate the sample size and geographical spread of the present survey in a similar one involving headteachers or pupils was, however, beyond the financial and logistical resources available to the researcher. It is hoped instead that the size of the sample of schools and teachers will help to compensate for the lack of triangulation in this element of the research. As will be discussed in Chapter 5, the numbers of teachers, from each of the primary stages P1 to P7 who responded, did in fact represent an acceptable spread.

Also, the return rate of 328 was sufficiently close to the figure of 333 derived from the formula proposed by Krejcie and Morgan (1970) to suggest that the number of responses received could be considered to be satisfactory for the purposes of this study. Krejcie and Morgan's formula was derived from one produced by the American National Education Association (NEA, 1960), for determining the sample size needed to be representative of a given population.

As noted previously this study was aided by a grant of £2000 from the Scottish Office Education and Industry Department. Whilst much of this money was used for printing and mailing costs, enough remained to allow the researcher to hire some part-time temporary help for data preparation and processing from returned questionnaires. The six students so employed for this purpose were briefed by the researcher before being asked to code the responses to the open-ended questions in the questionnaire. The coding categories had been provided by the researcher after he had carried out a content analysis of these responses.

Whilst it is normal practice in the analysis of qualitative data for responses to be encoded by at least two coders in order to establish an acceptable degree of intercoder reliability, in this instance the researcher was working alone and the responses given by the teachers were generally brief, unambiguous and straightforward.

#### 4.1.5 Summary timetable of events

November 1994	Pre-pilot exercise
January - May 1995	Pilot study
March 1996	Report of the pilot study
November 1996-February 1997	National survey
May 1997	Report of the national survey

### 4.2 The In-Depth Study

#### 4.2.1 Introduction

The findings of the national survey will be discussed in full in Chapter 5. One of the main results, however, should be mentioned at this point since it indicated the direction for the second and main part of this study. It became clear from the pilot study, and was subsequently confirmed in the results of the national survey, that teachers were very concerned with the implementation of the Problem Solving and Enquiry outcome of the *Guidelines* and that many of the questions they were asking could not readily be answered because of the lack of expertise and research evidence in this area. In fact, some of these questions had already been asked by the group of writers who produced the *Guidelines* - Review and Development Group 2 (RDG 2). Since the researcher had been a member of RDG 2, he realised how inadequate the *Guidelines* were in offering practical advice on planning, teaching, assessing and recording problem solving. For this reason it was decided to conduct a study into some aspects of children's acquisition of problem solving skills in mathematics. A particular focus of the study would be the ways in which

pupils acquire working knowledge of problem solving strategies and the role of metacognition in the development of these strategies.

Since one of the main focuses of this part of the study would be concerned with pupils' abilities to describe and report on their problem solving processes it was decided to work with pupils at the P7 (final year) stage of primary school. Section 2.2.7 has examined the difficulties of gaining direct access to the thought processes of learners, so with this in mind and using some of the research findings referred to in Section 2.2.7, it was felt advisable to work with older children who would be more likely to be able and willing to try to communicate and report on their processes and experiences. With these considerations in mind, the researcher decided to try to gain access to some P7 classes and was delighted, in the event, to find fourteen teachers who were willing to allow their classes to take part in the study. The process by which this was achieved is described in Section 4.2.3.

Section 4.2 contains brief sections on each of the following aspects of the study:

An overview of the methods. In this section the choice of methods is justified with reference to theoretical considerations identified in the literature.

Attracting, briefing, advising and providing support for participating teachers. Four sections of the chapter then explain how teachers were encouraged to take part in the study, how they were briefed about the programme by the researcher and given some advice about planning, organising and teaching a problem solving programme. Materials given to teachers by the researcher to support them throughout the year of the study, along with suggestions as to how they might be used, are described in the last of these sections.

Groups A and B. This section explains how and why two groups of classes were given slightly different requests about how one aspect of the programme might be implemented.

Design of the study. This section gives details of, and a timescale for, the actions which were planned for the rest of the year of the study, beginning with a pre-test, and which are described in greater detail in the next four sections of this chapter. These were:

- pupil observations and interviews
- pupils and teacher year-end interviews
- the post-test
- item pairs analysis.

#### **4.2.2 An overview of the methods**

In order to conduct such a study involving children, it was necessary to try to gain access to a number of P7 classes and their teachers who would be the subjects of the study during the course of one academic year. As noted previously, fourteen classes were identified to take part in the study. The timescale of one year was dictated by the fact that all the pupils in P7 at the beginning of the year would leave primary school and move to secondary schools the following August.

The study was primarily concerned with looking at the effects of a carefully planned and structured year-long programme of problem solving on pupils' acquisition of problem solving skills and attitudes. When planning the detail and structure of this programme, the researcher had taken account of the findings of others such as Schoenfeld (1985a) and Burkhardt (1988). As noted in Section 2.2.4 of this report, they had identified what they considered to be the necessary components of a problem solving programme for pupils. These were:

- frequent practice
- appropriate beliefs and attitudes

- an awareness of some common strategies
- the development of metacognitive skills.

The pupils in this study would be given regular and frequent practice of problem solving, the first of these four components, using a carefully compiled, structured and organised set of resources. It was hypothesised that the combination of the frequent practice, the organised use of the resources and the advice on teaching given to the teachers would generate the second component, the desired set of beliefs and attitudes in the pupils. The study would seek to determine the extent to which the remaining two components had been developed by the year-long programme and whether they could be more successfully developed by particular teaching actions.

To do this, it was decided to establish two groups, A and B, within the fourteen classes being used in the study and to give the teachers of the two groups different briefings with respect to some elements of their methods of teaching problem solving. In particular, the teachers of classes in group B were asked to take particular teaching actions which, it was hypothesised, might develop in their pupils a greater knowledge of problem solving strategies and might also improve their metacognitive skills. Teachers of group A classes were not given this request. The details of the requests made to the teachers of the two groups will be explained further in Section 4.2.7.

In designing this study, various aspects of triangulation were adopted. Methodological triangulation is referred to by Cohen and Manion (1994, p233) as, "the use of two or more methods of data collection", and "by making use of both quantitative and qualitative data". Other essential features of triangulation are that the methods are focused on the same topics and that the purpose of this is to increase validity and reliability of findings through cross-checking.

Pupils' problem solving behaviour and understanding were studied by the following data collection instruments:



- a pre-test and a post-test separated by an interval of nine months
- two sets of combined observations and interviews
- year-end interviews with pupils
- interviews with their teachers at the beginning and end of the year.

These provided elements of both methodological triangulation, since several methods were used, and time triangulation, since these took place over a twelve month period. In addition there was a degree of investigator triangulation, in the sense that information on pupils' problem solving skills and attitudes was being collected both by their teachers and the researcher.

For the study to be carried out over one academic year, the preliminary work had to begin much earlier, so in the Spring of 1996 fourteen teachers, who would be teaching P7 classes in the next academic session, were identified for the study. The teachers were then interviewed and briefed by the researcher and materials were prepared for the study. Initially all teachers were given the same briefing. The teachers of the classes who would constitute group B subsequently met and were given the additional briefing referred to above.

At the beginning of the 1996-97 academic year, the teachers were given all the materials necessary for them to take part in the study and their classes were given a pre-test in September 1996. Using the results of the pre-test, two pupils from each class were selected to be the subjects of further in-depth study for the rest of the year.

Each pair of pupils was observed and interviewed during a problem solving session on two occasions during the period from November 1996 to March 1997. They were then interviewed formally at the end of the year, in May 1997. In addition, a post-test was administered to all fourteen classes at the end of the school year. The penultimate part of the study was an interview with each of the participating teachers, again at the end of the year, in June 1997. The final data collection exercise, which took place in October 1997,

related to a technique designed to measure the amount of improvement in problem solving achieved by the classes involved in the study. This was an analysis of pupils' performance on pairs of test items, one of each pair being from the pre-test and one from the post-test.

#### **4.2.3 Attracting participants**

The first difficulty to be addressed by the researcher related to gaining access to a number of classes with which he could work. At a time when most teachers were feeling the stresses of implementing recent and major curricular changes, it was anticipated that few would be willing to volunteer to take part in a study which might involve them in even more work and interruptions to their normal classroom routine. On the other hand, the researcher was acutely aware of the fact that many teachers would appreciate some help in resourcing and teaching the problem solving component of their mathematics course. Accordingly it was decided that it would be tactically advisable to offer potential participants in this study a *quid pro quo* as an inducement to participate. This *quid pro quo* took the form of an offer to teachers to provide them with one year's supply of problem solving materials, presented in a structured and organised way, along with suggestions as to how these materials might be used to provide a complete set of problem solving experiences for a whole class, for the whole of their P7 year.

With a view to attracting teachers to the project, permission was sought and obtained from Angus Council to place a short article in their schools' newsletter describing the study and inviting teachers to apply to take part, on the understanding that they would be provided with materials to support them in their problem solving programme. This article appeared in the Spring edition of the Angus schools' newsletter and by the end of April 1996 requests had been received from thirteen P7 teachers to take part. At a later date, another request was received from a teacher in one of the pilot schools and, appropriate permission having been obtained, her class was also included in the study.

The response received from teachers was gratifying in the sense that no difficulty was experienced in attaining a viable number of participating teachers and classes. This meant, of course, that all the participating teachers were volunteers, with the possible distorting effects this might have on the research results. There was a possibility, for example, that they might all have been very concerned and lacking in confidence about problem solving. On the other hand, they may have been attracted to volunteer because of a special interest in the subject. To attend to these concerns, it was decided to explore each teacher's motives, attitudes and experience with respect to the problem solving and enquiry outcome of the *Guidelines*. This was done in the first instance during the first briefing session, described in Section 4.2.4 below, and subsequently during informal interviews with all teachers when they were visited individually in their schools by the researcher in June 1996.

#### **4.2.4 Briefing participating teachers**

The participating teachers were invited to the researcher's institution for a briefing meeting which lasted for a two-hour afternoon session during one of three in-service closure days for Angus schools. Each teacher was able to come on one of the three days and on each day approximately one third of the teachers attended. During these sessions the researcher was able to meet all the participating teachers and to describe the background and purpose of the study.

Each of the teachers was invited to describe where they were in the implementation of the *Guidelines'* problem solving and enquiry outcome. They were also invited to say what they hoped to get out of their participation in the research project. The information obtained from this informal survey was that most of the teachers had been trying to teach some problem solving since the implementation of the *Guidelines* in their schools two or three years previously. In most cases, however, these attempts had been lacking organisation and coherence, and the teachers felt that, up until this point in time, they had only been at the stage of dipping their toes in the problem solving waters. In common

with most teachers whom the researcher had met on in-service and staff development activities, they needed help in resourcing, planning, structuring, assessing and recording their problem solving activities. Most of them admitted modestly to a lack of expertise in teaching problem solving, but given that it had only recently been introduced into the curriculum as a result of the publication of the *Guidelines*, they were no different from most Scottish teachers in this respect. Some within the group expressed a special interest in the subject and all were motivated by a professional desire to do better than they had been doing previously. Some of them also felt that their schools would be looking to them to pass on to their colleagues some of the benefits of their participation in the project.

The researcher's own background as a member of Review and Development Group 2, which produced the *Guidelines* and his experience of having delivered many in-service and staff development sessions on the subject of problem solving, made him aware of the fact that most teachers were similarly lacking in confidence about how best to plan, resource and deliver a coherent problem solving programme. In this respect the participating teachers were no different from most teachers with whom the researcher had worked. The outcome of these discussions was to reassure the researcher that the group of teachers was not biased in any detrimental way and that the views they expressed were typical of those held by the several hundred teachers with whom the researcher had worked in the previous three years of problem solving in-service work.

After these informal discussions, the briefing sessions took the form of short presentations by the researcher, followed by question-and-answer and discussion sessions. The following topics were addressed:

- The background to the Problem Solving and Enquiry outcome, its components and rationale.
- The new and different roles demanded of both teachers and pupils in the implementation of this outcome.

- The difficulties facing teachers in coming to terms with and implementing this outcome, as shown by the results of the National Survey and numerous HMI school reports.
- The debate as to how children should best be expected to learn about problem solving strategies.
- The challenges of trying to identify appropriate levels within the Problem Solving and Enquiry outcome.
- Ways of providing progression and differentiation within the outcome.
- Methods of planning, organising, recording and assessing children's problem solving experiences.

At this meeting the researcher was able to persuade the teachers that he would try to provide help and support in all of the areas referred to above during the course of the study which would take place over the coming academic year, i.e. from August 1996 until July 1997.

#### **4.2.5 Support materials provided**

All participating teachers were subsequently given a bank of problem solving materials to use with their classes. The problems contained within this bank were all of the kind which the *Guidelines* described as being "tasks designed specifically to highlight the merits of certain approaches to mathematical thinking" (SOED, 1991, p48), or which Charles and Lester (1984b) describe more succinctly and arguably more intelligibly as 'process' problems. These were previously described in Chapter 2 as "self-standing, non-standard problems which have one identifiable solution, which can be solved by any one of a number of different processes or strategies and in which the process itself, as well as the solution, can be of interest". They also fit the descriptions offered by Goldin (1982) in his categories 2, 3 and 4, introduced in Section 2.2.2.

These problems were contained within six folios representing six problem solving strategies, with each problem being located in a folio according to the particular strategy the researcher felt would be the one used by most pupils to solve it. The six strategies were:

- look for a pattern;
- guess, check and improve;
- draw a table, diagram or picture;
- use logical reasoning;
- make an organised list;
- work backwards.

This was not seen as a definitive list of problem solving strategies and the teachers were urged to think of the use of these strategies in this context as an organisational device only. There would be many instances when children would use strategies other than the one expected for a problem in a particular folio and of course this was perfectly acceptable. The teachers were advised not to label the folios by the named strategy as it was the researcher's intention that pupils should become aware of the strategies through the dual experience of solving the problem and engaging in discussion about the processes used to do so. This inductive or experiential approach to developing an awareness of strategies had been arrived at after consideration of the arguments rehearsed in Section 2.2.4. The researcher's view was that pupils should learn about and be made aware of strategies explicitly, as advocated by many commentators, but should not be 'taught' them explicitly. Learning about strategies should follow experiences of using and, more importantly, describing strategies of which pupils will have had personal experience.

To help teachers allocate particular problems to children of all attainment levels in their classes, the problems within each folio were further categorised into one of three levels, designated by a circle for the easiest, a triangle for the more challenging and a square for the most difficult. Once again these were so designated according to the researcher's own

opinions based on his work in problem solving with a number of primary classes. Once again, teachers were encouraged to re-designate problems which their pupils' experiences suggested had been wrongly categorised.

The teachers involved in the study were also shown various ways of planning and recording their pupils' problem solving experiences. Each one was, however, asked to try to encourage their pupils to keep their own records of the problems they had solved. To this end all pupils were given a personal record sheet on which they recorded the name of each problem solved and the date on which it was done. There was a space at the bottom of each of these sheets on which teachers were encouraged to make formative assessment comments once per term. The sheets, and teachers' comments, could then be used for more formal year-end reporting purposes. A copy of the personal record sheet is in Appendix 4.2.

Teachers were also shown several versions of teacher-managed reports which the researcher had previously used with other teachers. Some of these were wall-mounted, some referred to groups and some allowed for teacher comments on each problem completed by individuals or by each group of pupils. No guidance was given as to which type the teachers in the study should use and this was left to their own discretion as assessment and teacher recording and reporting did not constitute part of the study. The only common reporting practice across all fourteen classes was the pupil self-recording sheet described in the previous paragraph.

The problems in the six folios were selected by the researcher from both a number of commercially available resources and some materials which had been produced especially for this study. The teachers were given a list showing the source of every problem and each problem was coded so that teachers could easily locate its source, its level of difficulty and the folio to which it belonged. Because of pressure of time in the first instance, but later because of the researcher's supposition that the teachers would be able to work with their pupils in solving the problems in the bank, no solutions were provided.

This decision was changed at the end of the study due to sustained pleas from a minority of teachers for the solutions.

#### **4.2.6 Advice given to participating teachers**

With the exception of one important aspect of the study which will be discussed later, all teachers were given the same advice on how the problem solving programme of work should be delivered. The teachers were all working with the same materials, which were all structured and organised in the same way, and all the children involved in the study were using similar reporting formats.

In addition, the researcher asked the teachers to spend approximately thirty to forty five minutes per week working on the materials and all were asked to observe these limits. None of the teachers foresaw any difficulty in so doing. This advice was given to the teachers in their initial briefing meeting and was repeated in a set of written instructions which accompanied the problem solving bank sent to their schools in August 1996. These instructions are given in Appendix 4.4. Previous to this all the teachers had been visited and interviewed personally by the researcher in June 1996 to collect further information on their personal approaches to mathematics teaching in general and to their teaching of problem solving in particular. The interview schedule used to find out about their teaching of problem solving is shown in Appendix 4.5. They had previously described their progress in problem solving during the briefing session they had attended in May or early June 1996. In the majority of cases, as noted earlier in Section 4.2.4, they had done little or no problem solving in any organised way and it was this fact which had prompted most of them to sign up to be part of the study in the first place. At these interviews information was obtained from each teacher on the following:

- The number of pupils who would be in their class for the coming year and the numbers among these who would be working towards level D in mathematics.



- Teachers' willingness to have their class work on problem solving for the agreed amount of time each week.
- Any problem solving resources used previously and still available to the teacher.
- Any existing organisation or structure for a problem solving programme.
- The amount of teaching or intervention which they would anticipate using in the delivery of their problem solving programme.
- The extent to which they had made use of the Starting/Doing/Reporting cycle when teaching problem solving.
- Whether they had previously taught their pupils about strategies and if so how.
- How their pupils reported on their problem solving.
- Number of years of teaching and number of years of teaching problem solving.
- Class organisation used for problem solving.

With respect to the last of these topics mentioned above, some time was spent with the teachers discussing the pros and cons of different organisational models which they might use when their classes were involved with problem solving. In the course of these discussions it became clear that each teacher would be most comfortable with the existing mathematics organisation used in his or her class. The most common structure used was for pupils to work on problem solving in groups, but this was not seen as group problem solving as most teachers wanted their pupils to have the satisfaction of completing as many problems as possible on their own. In fact most pupils worked individually within groups and it was common for several different problems to be attempted within one group. This model did allow for some cooperative work to be done, but this cooperation tended to be done in pairs rather than in groups. Some discussion took place as to the respective merits and drawbacks of cooperative group work as discussed in Section 2.2.9 of this report and it was agreed that each teacher would use the organisational structure with which he or she was most comfortable. All teachers did, however, agree that every pupil should be responsible for completing the solutions in writing on their own and recording these solutions in their own folders.

The interviews also provided a further opportunity for teachers to speak with the researcher on an informal and individual basis and to have any questions about the study answered.

4.2.7 Groups A and B

The information gathered from the initial meeting and individual interviews with the teachers furnished the researcher with sufficient information to establish two separate groups within the fourteen classes involved in the study. This was done in an attempt to set up what Cohen and Manion (1994, p168) refer to as a 'quasi-experimental design'. The designation of 'quasi' experimental refers to the impracticality of using a totally random selection of pupils or classes. The model used was one which at first sight seemed to fit Cohen and Manion's description as illustrated by the diagram in Figure 4.1:

Figure 4.1  
*Cohen and Manion's quasi-experimental design diagram*

Experimental	O1	X	O2
Control	O3		O4

This figure uses the conventions of Campbell and Stanley (1963). 'O's refer to the processes of observation or measurement and the 'X' represents the exposure of the experimental group to a variable or event, the effects of which are to be measured. In this case O1 and O3 represent the pre-test given to both groups and O2 and O4 the post-test. The X event was intended to be an additional set of experiences to which half of the classes were to be exposed.

On reflection, however, this model was seen to be inadequate to describe the design of this study, since both groups of classes were going to be subjected to a variable which they had not previously experienced, namely the complete problem solving programme for the year. The classes which had initially been thought of as comprising the experimental group were going to be given an additional and different experience or variable. Since both groups were to be exposed to new events, it was not appropriate to

refer to one as a control group in the normally accepted sense of the word. In this case a control group would have had to consist of a number of classes following their normal P7 curriculum which would probably have included some problem solving activities. For this reason it was decided not to refer to the two groups of classes as control and experimental, but as group A and group B. Figure 4.2, which is a modified version of Cohen and Manion's, offers a better model of this study.

Figure 4.2  
*Modified quasi-experimental model used in the in-depth study*

Group A	O1	X1		O2
Group B	O3	X1	X2	O4

X1 refers to the programme of problem solving to be experienced by both groups of classes and X2 refers to the additional experience which seven of the fourteen classes would have. The effects of both experiences X1 and X2 would be measured throughout the year. O1 - O4 remain as defined above.

In this case, fourteen classes were divided equally to form the two groups A and B. These were selected in such as way as to ensure that both groups had a similar balance between types of schools and classes - large and small, urban and rural, composite and single-stage. No account was taken of the proportion of boys and girls in each class. Only two criteria were used for selecting classes for inclusion in the group B. These related to their teachers' previous practice in teaching problem solving. Three of the fourteen teachers said that they encouraged their pupils to report on the processes used to solve problems, either in a written or oral form and another one had made some overt attempts to encourage her pupils to learn some problem solving strategies. These four teachers were included in group B, since teachers in this group were going to be encouraged to do what these teachers had already begun to do independently. The remaining ten classes were divided among the two groups to give each group the kinds of balance referred to above, i.e. size of class and type of school. Information about the schools is given in Appendix 4.3.

Before the two groups had been selected each teacher had received identical information and briefing. When the two groups referred to above were defined the teachers of group B classes were given additional written information. This took the form of an amended version of the instructions given earlier to the whole group of teachers (Appendix 4.4). This new version, (shown in Appendix 4.6) had an extended section of notes on 'teaching'. In this, they were asked to place more emphasis on the 'reporting' phase of the problem solving cycle and to encourage their pupils to provide a written report on every problem they completed. To facilitate this, the teachers of group B classes were given self-reporting forms very similar to those already shown to all the teachers but containing an additional column to be completed by the children, entitled "How I solved it", (Appendix 4.7). This addition was a conscious effort to force the children to think about, analyse and describe their own thought processes. The researcher's previous experience with children of similar ages to those in the study had made him acutely aware of how difficult a task this would be initially for pupils, even at the P7 stage, so he was able to brief the teachers to the effect that they should not expect their pupils to be very skilled at reporting on their processes when they were first asked to do so. This additional set of instructions was the 'X2' event shown in Figure 4.2.

The purpose of defining two groups in this way was to investigate whether continued weekly practice and experience of reporting processes used would lead in the course of the year to a greater awareness of the range of strategies used. The extent to which the continued emphasis on reporting was helpful in developing a knowledge of strategies would also be explored.

There are, however, a number of well-established threats to the internal validity of experimental treatments, such as that proposed for group B, over which the experimenter has little or no control. An experimental treatment is said to have internal validity if the experimental treatments do in fact make a difference in the experiments being studied. Common threats to internal validity include the following:

- Maturation effects. Subjects of a study can mature between any two measurements in ways which are independent of experimental treatments. The longer the time interval between measurements, the greater the likelihood there is of this happening.
- History or non-natural change. Events other than the planned treatment can often happen between pre- and post-tests and these events can produce effects which can mistakenly be attributed to experimental treatments.
- Statistical regression. Regression to the mean can occur between pre- and post-tests.
- Other difficulties relate to testing, instrumentation and selection bias.

The design used sought to minimise these risks to the study's internal validity. Tests were piloted with a view to improvement, classes were selected and assigned to groups A and B as previously explained, teachers were briefed explicitly and materials were provided for all pupils to work with throughout the year. The researcher of course had no control over teacher behaviour beyond the suggestions he had made to all participants and the agreement he had with the group B teachers to try the additional reporting activity described. Teachers in group A might, for example, independently arrive at a similar or alternative way of encouraging reporting. The researcher would try to discover as much as he could about possible clouding conditions during informal conversations and formal year-end interviews with the teachers.

#### **4.2.8 Design of the study**

At the beginning of the academic year 1996-97 a pre-test was administered to all 237 pupils in the classes involved in the study. The test had been piloted with two P7 classes in schools not involved in the study and adjustments had been made to some of the items in light of the findings of the pilot study. Teachers were asked to administer the six-item pre-test in two parts of three items each, using a specified length of time and with a week between each session. The test was given to all the pupils in the classes which would be

taking part in the problem solving programme for which the researcher had provided the materials. A copy of the test is in Appendix 6.1. The six items in the test are similar in nature to problems which pupils would meet in commonly used textbooks and to those given to each of the participating teachers, and which their classes would be working on throughout the year. The common factor in all the problems used, was that they should engage the pupils' interest and that pupils would be sufficiently motivated by the content to want to solve them.

As soon as the pre-test had been completed and returned for marking to the researcher, the classes began their year's programme of problem solving. When the tests had been marked, a small number of pupils in each class whose scores fell within a defined band were identified by the researcher as those from whom two pupils would be selected for in-depth study during the year. Since the study was to involve observations and interviews conducted by the researcher working alone, it was felt that two pupils from each of the fourteen classes would be the maximum number who could be studied, observed and interviewed at any one time.

The maximum possible score on the pre-test was 32 and it was decided to select pupils scoring between 20 and 27 for the subsequent in-depth study. This band did not represent the highest scoring pupils in every school since there was a small number who scored more than this. It was decided, however, not to include these exceptional cases and to select from the 20-27 band since it seemed to represent a range of ability which was present in most of the classes. In most classes there were also sufficient pupils scoring within this band to allow for some selection of pairs of pupils. In the event it proved impossible to apply this criterion to all classes since, in some of the smaller rural schools with very few pupils working at level D in mathematics and in one or two of the poorer classes in the bigger schools, there was either one or no pupil achieving such scores. In response to this problem it was necessary to broaden this band below 20.

When the results of the pre-test were returned to the teachers, the names of all children scoring within the designated band were highlighted and the teachers were asked to identify, from those pupils, two whom they thought would respond willingly and would be able to articulate their thoughts reasonably well in a series of interviews and problem solving sessions conducted and observed by the researcher. In two of the very small rural schools referred to above, nobody scored within the initially required band so two pupils who narrowly missed it were chosen. In three other schools, only one child fell within the initially required range so the second was again chosen from among those narrowly missing it. It was considered important to have two children from each class to avoid feelings of isolation which might have resulted from only one having been selected. The selection was left to the teachers since it was felt that they would be able to choose two pupils whose ability to express themselves would make the interviews more productive. At this stage the researcher was acutely aware of the difficulties attached to eliciting reliable verbal reports which could be used as data. As noted previously in Section 2.2.7, some researchers such as Lester (1987) and Andrews (1992) had noted and commented on these difficulties, whilst others such as Ericsson and Simon (1980) and Lawson and Rice (1987) had suggested ways of overcoming some of the shortcomings of verbal reporting. In their 1995 study, Desforges and Bristow adopted strategies similar to those described here, to try to elicit and maintain pupil verbalisation which could be used as data.

Three observation and interview sessions were carried out with each pair of pupils. These took place in December 1996, March 1997 and June 1997. On the first two occasions the children were observed and questioned during a problem solving session and on the final occasion they were interviewed about their problem solving experiences.

In May/June 1997 all pupils involved in the study completed a post-test and all the teachers involved were interviewed

#### 4.2.9 The pupil observations and interviews

These sessions were conducted to collect data relating to children's metacognition - their ability to monitor and regulate their own cognitive processes - and to assess the extent to which they were aware of some common problem solving strategies.

To do this it was necessary to devise a framework for observation of the pupils' problem solving. Consideration was given to frameworks similar to those described in Section 2.2.5 and those used by others such as Schoenfeld (1987b) who worked with college students and Lester, Garofalo and Kroll (1989) who used their framework to design and observe research tasks for grade 7 students. The framework chosen for this study was the three-part scheme identified in the *Guidelines*, i.e. Starting, Doing and Reporting. It was considered preferable to use a framework such as this for two reasons. Firstly, it was not entirely unknown to teachers and, from the evidence gathered during initial interviews and briefing sessions, they were reasonably confident that they understood and could use it. Secondly, having only three 'episodes', it was simpler than most of the others referred to earlier and hence would be easier to use for observation, recording and analysis purposes by the researcher working as an individual. Most of the other frameworks described in Section 2.2.5 used more than one observer and reporter.

Schoenfeld's six element framework for protocol analysis, described in Table 2.1 and in Figures 2.1 and 2.2, was tried by the researcher in the first two sets of pupil observations and interviews conducted, but was found to be too complex and unwieldy to use. It was impossible, by observing children working in silence, to distinguish, for example, between the elements, 'Read', 'Analyse' and 'Explore'. Since the children in this study were not asked to think aloud, but were invited to report concurrently by responding to the interviewer's interventions, there appeared to be no way of telling when each episode ended and the next one began. Even if the pupils had been asked to use think-aloud techniques, it is very doubtful whether, as P7 pupils, they would have been able to distinguish between the different mental activities which constitute the episodes which

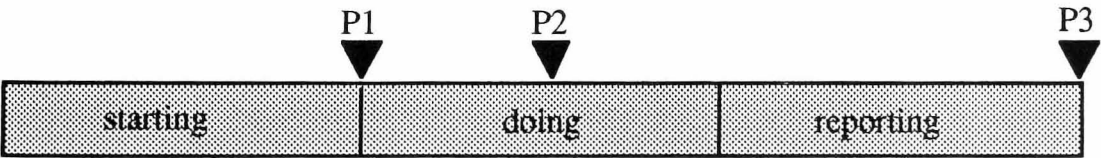


Schoenfeld had used with college students. The logistical challenge of working with two pupils simultaneously whilst implementing the observations/intervention model described below, further served to convince the researcher of the impossibility of operationalising Schoenfeld's model. This decision was confirmed by the advice of Green and Gilhooly (1996) to think carefully about the feasibility and the practicability of collecting protocols.

Two of Schoenfeld's (1983) ideas were, however, conjectured to be helpful in the application of this framework. These were his 'episodes' which he described as "periods of time during which the problem solver is engaged in a single set of like actions", (p 347) and his 'executive decision points' which occur between each episode and are instances where managerial decisions of a metacognitive nature must be taken. In the framework used in this study there were, in general terms, three broad episodes - the Starting, Doing and Reporting stages of the framework. The relevant executive decision points occurred before and after each of the first two episodes.

In each of the observation sessions the two pupils selected from each class were invited to solve two problems working independently and without collaboration. At the executive decision point between the 'Starting' and 'Doing' episodes (P1), at some point (P2) during the 'Doing' phase, and at the end of the problem (P3), they were invited to respond to an instruction or probe by the researcher, as illustrated in Figure 4.3 below.

Figure 4.3  
*Graph showing pupil observations and intervention points*



Before each session began, the purpose of all the researcher's questions had been explained to the pupils and they were invited to ask any questions they might have had. The first episode of reading and digesting the problem, or trying to assimilate it to a previously met type, is referred to as the 'Starting' element of the framework. This episode was usually characterised by silent reading, interpretation of the conditions of the

problem and thought about possible approaches to be used. When it became clear from observation of each pupil that he or she was about to begin the 'Doing' phase of the framework, by putting pencil to paper, the researcher made the first intervention or probe by asking, "Can you tell me what you think you will do?". This is referred to as P1 in the diagram above.

When the pupil had been working on the problem for some time and was, on the basis of the researcher's observations, clearly involved in the 'Doing' phase, a second intervention was made. This time the researcher asked, "Can you tell me where you are, how you are getting on and what you are going to do next?" This probe, P2 on the diagram, was intended to elicit a response which would show the extent to which pupils were aware of the cognitive processes they were using and the extent to which they were able to regulate or control the direction which should be taken. The question was deliberately composed in three parts. The first invited pupils to describe the stage they had reached in the mechanics of the solution process. The second attempted to elicit a subjective judgement or evaluation of the work they had carried out on the problem up to that point, and the third sought to determine the amount of regulation of their own processes they were exercising. Both the P1 and P2 probes were attempts to elicit some think-aloud data and some concurrent reporting, in the terms discussed in Section 2.2.7. By avoiding the need for subjects to recall from their long term memories, it was hoped that the pupils in this study could, similarly to those in Goos and Galbraith's (1996) study, provide good samples or descriptions of their cognitive processes. The last probe, P3, used at the end of the final episode, invited a retrospective response which, it was hoped, would provide some evidence of pupils' awareness of strategies used and their ability to describe these strategies. P3 asked pupils to respond orally to the question, "What was it that you did that allowed you or helped you to solve this problem?". By making this probe as soon as the subjects had completed their solutions, it was hoped to minimise the need for recall from LTM.

At this stage the work of Smith and Miller (1978) was tacitly accepted by the researcher. They, amongst others, had rejected the premise of Nisbett and Wilson (1977), that students had no access to some of their most relevant higher cognitive processes. Smith and Miller's contention was that it is profitable to look for accurate verbal reports of mental processes by students provided they are working on tasks which they find engaging. Also, by inviting responses to probes which refer to information that the student is currently attending to and which do not impose heavy or confusing demands on their memory, it was expected that the resulting verbalisation would not affect too adversely, or might even assist, their cognitive processes (Ericsson and Simon, 1980; De Corte and Verschaffel 1987).

The researcher was also aware of the limitations of verbal accounts as data and the rather different view taken by Lester *et al.* (1989b), who noted that "for some students, thinking aloud during problem solving was unnatural and sometimes had a debilitating effect on their performance." (p119). Also, a possible alternative, i.e. retrospective reports from long term memory may be unreliable or subjects' verbal descriptions of their cognitive processes may be inconsistent with behaviour observed by the researcher (Goos and Galbraith, 1996). Strategies used by the researcher to overcome these reservations included making the tasks interesting and motivating, keeping them of relatively short duration, seeking to reduce dependence on LTM by the use of concurrent probes and on the one occasion where a retrospective probe was used, making it immediately consequent to the completion of the solution. All the pupils' responses were audio-taped for future coding and analysis.

Taking into account the factors mentioned above relating to verbal data, as well as the age of the pupils and the researcher's own experience of the difficulties of attempting to persuade students to articulate or describe their own thought processes, the initial conjecture in this study was that pupils' metacognitive abilities, or their ability to articulate these, or both would be poor. Nisbet and Shucksmith (1986, p7) remark that,

To speak of metacognition....., as a sophisticated awareness of one's mental processes, seems to place it far beyond the capacity of primary school children.

They subsequently conclude, however, that,

children already begin to develop metacognitive knowledge or awareness..... while they are still in the primary school.

This view supported the researcher's decision to include a study of the metacognitive skills of primary seven pupils in this research. Schoenfeld (1985), on the other hand, said that his own university students' metacognitive skills were poor since virtually all mathematics teaching to which they had previously been exposed, had focused on mastering facts and procedures involving low-level skills and unreflective use of rules.

The study described here sought to explore the extent to which improvement in metacognitive abilities over the course of part of one year (primary seven), could be noted and measured. To this end, this first set of observations was designed to explore, at the beginning of the year's structured programme of problem solving, the extent to which pupils in both groups A and B were able to demonstrate their abilities to control, describe and analyse their cognitive processes during mathematical problem solving and to identify which strategy they had used to reach the solution.

This interview and observation process was repeated in March 1997 and the responses to the intervention questions and prompts were coded in the same way, to ascertain whether children's responses to probes were becoming more analytical and to look for more evidence of metacognition in their answers than there had been in the November 1996 responses.

#### **4.2.10 Pupil and teacher year-end interviews**

In June 1997 all the pupils who had been involved in the previous interview/observation sessions were interviewed individually by the researcher. These interviews sought to

examine pupils' perceptions of what they had learned in general terms about problem solving and what problem solving behaviours they had learned, including the use of strategies. Their attitudes to problem solving and the difficulties they had encountered were also explored as were their reporting practices.

All fourteen teachers involved in the study were also interviewed in June 1997. The purpose of these interviews was to explore the teachers' perceptions of the success or otherwise of their problem solving programme over the year. They were also asked about their pupils' learning with particular reference to strategies. Other questions were asked about processes of assessing, recording, reporting and organisation. The results of both sets of interviews are discussed fully in Chapter 6.

#### **4.2.11 The post-test**

A six item post-test was given to all 237 children in the study at the end of May 1997. The test was similar in structure and content to the pre-test which the children had done at the beginning of the year. Direct comparisons of performance on the tests overall and on individual pairs of items would permit some quantitative comparisons of students' work to be carried out and the performance of schools and pupils in each of the groups A and B to be studied.

#### **4.2.12 Item pairs analysis**

The last piece of data collection, done in October 1997, was conducted by the researcher to enable measurements to be made of improvements in pupils' performance on problem solving between the beginning and end of the year during which the study was conducted. To do this, pairs of questions were selected, each pair consisting of one question from the pre-test and one from the post-test. These pairs of questions were chosen by the researcher as being those which, in his experience and judgement, represented problems which were most likely to be solved by most of the pupils in the

study using the same strategy. For example, question 1 in the pre-test was paired with question 1 in the post-test since they were very similar in structure and depended for their solutions on the children reading the clues, and by use of logically deductive reasoning, arriving at their solution. Six pairs of questions were formed in this way.

Each of the question pairs thus selected was then given to one of six different P7 classes in October 1997. The pupils in these classes had not previously been involved in this study and had done little problem solving up till that point in their P7 year. The teachers of these classes, whose agreement to undertake this small task had been obtained by the researcher, were given written instructions on how these pairs of questions should be presented to their classes. They were asked to have half of their class do the second question first, followed by the first one whilst the other half attempted them in the normal order. This was an attempt to minimise the appearance of any possible 'fatigue factor' which might have reduced pupils' performances on the second question and alternatively any 'warm-up factor' which might have resulted in enhanced performances on the second question.

The pupils' efforts were then marked by the researcher. Each of the two questions in the pair was assigned the same maximum mark and the pupils' work was graded using similar marking schemes. The scores gained on each of the two questions in every pair allowed a comparison of difficulty to be established between the two items. For example, comparing the first pair of items, the average score on item 1 from the pre-test was 81.9%, whilst that of item 1 from the post-test, done by the same class at the same time, was 89.6%. This provided a comparative difficulty factor of 1.09 to be used to compare these two items. Similar comparative difficulty factors were found for each of the six pairs of items. These factors allowed the researcher to calculate, on the basis of the scores on each item in the pre-test, an expected score on the corresponding item in the post-test, for all pupils who had been involved in the year-long study. Any improvement on this expected score in the achieved post-test scores would represent the 'added value' attained during the pupils' P7 year. This is discussed further in Section 6.3.

### 4.2.13 Links between method and research questions

A summary of the links between the research questions and the methods and instruments used to address them is given in Table 4.1

Table 4.1  
*Links between method and research questions.*

Research question	Addressed by:	Chapter 4 section:
RQ 1	Survey questionnaire	4.1.4

Comments: The national survey by written questionnaire was the only instrument used as it was seen as the most efficient way in which one researcher could collect the data required.

RQ 2	Survey questionnaire	4.1.4
------	----------------------	-------

Comments: The same instrument was also considered the best means of identifying common views shared by teachers throughout the country.

RQ 3	Teacher and pupil interviews	4.2.10
------	------------------------------	--------

Comments: The two sets of interviews conducted at the end of the year, were intended to collect the retrospective reflections of all the participants about the programme.

RQ 4(a)	Pupil observation and interviews	4.2.9
---------	----------------------------------	-------

Comments: The observation and interviews conducted during pupils' problem solving sessions at intervals during the year, were intended to find evidence of pupils' growing metacognitive awareness.

RQ 4(b)	Pupil observation and interviews	4.2.9 & 4.2.10
---------	----------------------------------	----------------

Comments: Pupils' developing knowledge of strategies would be observable in their descriptions of their own processes during the observed sessions and in the number of strategies which they could report having used at the end of the year.

RQ 4(c)	Pre- and post-test comparisons & item pair analysis	4.2.11 & 4.2.12
---------	---	-----------------

Comments: The growth in problem solving skills over the course of the year was measured by comparing the results of the pre- and post-tests. The item pairs analysis provided a vehicle for measuring the actual improvement against an expected performance and for comparing improvements registered by different groups of pupils.

RQ 5	Observations, interviews, pre- and post-test	4.2.9 - 4.2.12
------	--	----------------

Comments: The data collection instruments listed here, were used to look for evidence that 'reporting' resulted in improved performance sufficient to justify its inclusion as an aspect of a problem solving framework.

RQ 6	Teacher and pupils interviews	4.2.10
------	-------------------------------	--------

Comments: It was hoped that both teachers and pupils would make reference to positive changes in the affective domain, although the pupils were not asked explicitly about their attitudes, beliefs and feelings.

---

#### 4.2.14 Summary timetable of events

March 1996	Request asking for volunteer teachers
April 1996	Participating teachers identified
May 1996	Briefing meetings with teachers
June 1996	Individual interviews with teachers
September 1996	Pre-test administered
October 1996	Problem solving materials given to schools
October 1996	Pairs of pupils identified in each class
November 1996	Pupil observations/interviews
March 1997	Pupil observations/interviews
May 1997	Pupil interviews
June 1997	Post-test administered
June 1997	Teacher interviews
October 1997	Question pairs data collected



### 5.1      Introduction

This chapter reflects the content and sequence of the questionnaire used to collect the data. Although normal convention would indicate that this chapter should contain the results of the survey with a following chapter devoted to a discussion of these results, it was felt that in the interests of clarity and continuity some discussion of the findings should accompany each section in this chapter. Each section of the chapter therefore has three parts - a summary of the findings, followed by the detailed results and finally a commentary on the findings. A more general discussion of the results and their implications will be given in Chapter 7.

As noted in the previous chapter, seven areas were chosen for this national survey. These were:

- teachers' views of the *Guidelines*
- effects of the *Guidelines* on classroom practice
- support available to help implementation
- teachers' views on the Problem Solving and Enquiry outcome
- the specific issue of calculating
- the use of microcomputers in primary mathematics
- the use of context in mathematics.

In addition it was decided to ask the teachers in the survey about:

- their feelings about mathematics
- their own levels of attainment in the subject
- the attitudes of their colleagues in school to the *Guidelines*.

The questionnaire used in the survey is in Appendix 4.1.

The results discussed in this chapter are based on returns from 328 teachers in 195 primary schools. These figures represent a 36% response rate from the teachers surveyed and a 50% response from the schools surveyed.

Of the 328 questionnaires returned, 31 were received from schools with rolls less than 70, 36 from schools with rolls between 70 and 150 and 261 from schools with rolls greater than 150. Table 5.1 below shows the percentages of participating teachers who taught at each particular primary stage. Since many of the teachers in the sample taught more than one stage, the 311 teachers who responded to the relevant question (Q36) taught a total of 493 stages, so the percentages given are of 493.

Table 5.1  
*Primary stages taught by teachers in the survey. (n=311)*

Stages	P1	P2	P3	P4	P5	P6	P7
Percentage of participating teachers at each stage.	13.8	13.4	10.5	13.8	13.2	15.2	20.1

Further analysis of data obtained from the responses to question 36 showed that 99, or 31.8% of these 311 teachers taught composite classes consisting of from two to seven primary stages. The results of separating teachers of composite classes from those teaching only one stage are as shown in Table 5.2.

Table 5.2  
*Primary stages taught. (n=311)*

Stages	P1	P2	P3	P4	P5	P6	P7	more than one stage
Numbers of teachers	35	27	19	26	28	24	53	99
Percentages of teachers	10.7	8.2	5.8	7.9	8.5	7.3	16.2	31.8

### 5.1.1 Summary of the findings

The following are the major themes which have emerged from this survey.

- There was a generally positive acceptance of the *Guidelines* by the vast majority of teachers.
- The framework of outcomes, strands, targets and levels had led most teachers into a search for new formats for assessing, recording and reporting mathematics. Most saw the 5-14 framework as a positive help in these areas.
- Very little resistance to change *per se* was noted and relatively few complaints were received from teachers about the implementation of the *Guidelines*. There was, however, a plea for more time to adapt to, to come to terms with, and to implement the ideas in them. A period without change for the foreseeable future would be much appreciated by most teachers.
- The Problem Solving and Enquiry outcome was a source of concern for many teachers and help was requested with issues relating to levels, progression, assessing, recording and reporting within this outcome. In addition, teachers felt a need for clarification about how problem solving strategies were to be made explicit to their pupils.
- Teachers seemed to be in agreement with the recommendation to reconsider the emphasis placed on each of the modes of written, mental and calculator calculations, but were making actual changes only very slowly.
- There was a perceived need for extra resources, especially those designed explicitly to support the *Guidelines*.
- There was a continued growth in the use of microcomputers in mathematics but teachers felt hampered by insufficient numbers of machines, lack of familiarity with recent software and applications and felt in need of staff development in this area.
- Teachers welcomed the move towards greater contextualisation of mathematics, but had reservations about their ability to put these recommendations into practice.

## 5.2 Findings of the survey

### 5.2.1 Teachers' views of the *Guidelines*

This brief initial section of the questionnaire tried to establish at what stage of implementation of the *Guidelines* teachers were, their views of the *Guidelines* and the reasons why these views were held.

#### Summary

The main points found were that:

- most teachers felt that they had completed or nearly completed the implementation of the *Guidelines*
- most viewed the *Guidelines* positively, with 80% saying they liked them either 'a little' or 'very much'
- the *Guidelines* were seen as making sensible recommendations, trying to relate mathematics to real life and as having realistic attainment targets.

#### Detailed findings

The data in this section, collated from the responses to questions 2, 3 and 39, indicated a high level of approval of the *Guidelines*. As noted above, the responses to question 2 showed that 80% of teachers liked them, with only 7% expressing any degree of dislike.

These figures were confirmed by the more detailed probing used in question 3, in which teachers were asked to tick the box which most nearly matched their own views on a number of statements about the *Guidelines*. Table 5.3 shows the results in percentages.

Table 5.3  
*Teachers' feelings about the Guidelines. (n=328)*

	strongly agree	agree	disagree	strongly disagree	no response	% agreement	% disagreement
The Guidelines make sensible recommendations	16.8	75.6	3.7	0.6	3.3	92.4	4.3
They are easy to read	7.3	66.2	20.7	1.2	4.6	73.5	21.9
They attempt to relate maths to real life	7.3	79.3	7.6	0.6	5.2	86.6	8.2
They encourage children to use and apply maths to a greater extent than before	10.4	56.4	25.3	2.7	5.2	66.8	28.0
The targets are realistic	8.2	74.7	11.9	0.6	4.6	82.9	12.5

Whilst it is clear from these figures that most teachers agreed with all of the given statements, there are two statements which did not elicit the same level of agreement as the other three. Almost one quarter of the sample did not agree that the *Guidelines* were easy to read and more than a quarter disagreed with the statement that they encouraged children to use and apply mathematics to a greater extent than before. A cross tabulation of question 1, about the stage of implementation of the *Guidelines* reached, with each of the statements in question 3 (Appendix 5.1), shows clearly that teachers' agreement with each of the statements was related to the stage of implementation they had reached. Although roughly equal numbers (approximately 42%) were at each of the 'nearly complete' and 'complete' stages of implementation, in every case there were more respondents 'strongly agreeing' with the statements among those who had completed implementation than there were among those who had 'nearly completed' it. There was also a high correlation, at the 99% confidence level, between question 2 and all parts of question 3. (Appendix 5.2)

Other reasons given in the open part of question 3 for liking the *Guidelines* included the beliefs that they:

- ensured consistency of practice throughout the country
- provided clear progression and balance
- were helpful for planning
- provided a broader mathematics curriculum
- encouraged a more structured approach to mathematics
- encouraged greater use of context
- provided a worthwhile course which made them one of the easiest sets of guidelines to implement.

It was interesting that the 7% minority who did not like the *Guidelines* provided many more reasons to support their beliefs than the majority who liked them. Reasons given for not liking them included the following:

- Too much change was being sought too soon.
- There was too much 'jargon'.
- More time was needed for implementation.
- More resources were needed.
- There was a mismatch between existing textbooks and the *Guidelines*.
- More in-service was needed.
- Secondary schools were ignorant of, or uninterested in the *Guidelines*.
- Teachers were unclear about the problem solving and enquiry outcome.
- The targets were too low.

None of these reasons was given by many teachers. The most commonly occurring one, which related to the need for more resources, was mentioned only seven times, so they should not detract from what was an overwhelmingly positive response to the *Guidelines*.

### Commentary

The overall reactions to the *Guidelines* were encouraging and should be welcomed by curriculum developers since they demonstrated a broad acceptance of the changes being

mooted. This acceptance was, however, tempered by an awareness of classroom realities, manifested by the repeated requests made in the open response part of question 3 and in question 39, for more time, resources and support. As a member of the Review and Development Group (RDG 2) which produced the *Guidelines* the researcher was slightly concerned with the figure of 28% who disagreed with the statement that the *Guidelines* encouraged children to use and apply mathematics to a greater extent than before. This had been one of the underlying and unexpressed goals of the *Guidelines* which lay behind the introduction of the Problem Solving and Enquiry outcome and the idea of promoting greater use of context in mathematics. RDG 2 had intended that mathematics, following the implementation of the *Guidelines*, would now be seen as more than simply a 'toolkit' of facts, concepts and techniques and that teachers and pupils would now have a greater awareness and appreciation of the subject's application, relevance and usefulness in a variety of real life contexts. With 28% of teachers failing to pick up this message, which, with the benefit of hindsight, the researcher now feels should have been articulated more explicitly, there was a worry that a corresponding number of pupils would similarly fail to acquire this appreciation.

### **5.2.2 Effects of the *Guidelines* on classroom practice**

#### Summary

This section investigated five areas of change which had been identified by participants in the pilot study. Teachers were asked to say whether 'minor', 'major' or 'no change at all', had taken place within each of five areas and to specify briefly what these changes were. In four out of the five areas, between 85% and 90% of teachers felt that changes had taken place. The exception to this concerned the way in which mathematics was taught, which a smaller number (70%) felt had changed.

The five areas of change discussed were: content taught, methods of teaching, approaches to planning, assessment and recording.

Detailed findings

Table 5.4 shows the percentage of those responding under each heading to the request in question 4:

*"Please indicate the effects of the Guidelines on each of the following aspects of your work in mathematics."*

Table 5.4  
*Effects of the Guidelines on teachers' work in mathematics. (n=328)*

	no effect at all	minor changes	major changes	no response	% identifying change, major or minor
The mathematical content you teach	6.4	78.4	11.6	3.7	90.0
The ways in which you teach mathematics	26.2	62.2	7.6	4.0	69.8
The ways in which you plan your mathematics teaching	7.9	44.5	42.4	5.2	86.9
The ways in which you assess pupils' work in mathematics	9.8	46.6	39.3	4.3	85.9
The ways in which you record pupils' mathematics attainment	5.8	33.8	56.1	4.3	89.9

It is clear from these results that changes had occurred in all five of the areas mentioned, with the smallest amount of change, in relative terms, perceived to be in the way mathematics was taught. This is hardly surprising since the *Guidelines*, as noted above, did not have an explicitly stated purpose of conveying messages about teaching methods.

The greatest degree of change was felt to have taken place in recording pupil attainments, with most responses identifying this aspect also as the one in which most major changes had taken place. The changes occurring in each of these five areas are now described in more detail. These data come from teachers' responses to the open questions 5(a) to 5(e).

The single biggest change, in terms of the **content** which teachers were being required to teach, arose from the introduction of the Problem Solving and Enquiry outcome. This was identified by more than a quarter of all teachers who completed the questionnaire, although most of them described these changes as minor.



The second biggest change referred to (by 15% of teachers) was in the greater range of topics covered in the *Guidelines*. Ten percent of responses mentioned the fact that there was less arithmetic and a similar number referred to the greater amount of IT than previously had been the case.

Other content changes, with percentages of respondents mentioning each in brackets, were:

- the introduction of Information Handling (6%)
- the lower expectations of pupils (5%)
- the adoption of a new textbook series (5%)
- more practical work (4%)

Little or no reference was made under this heading to the suggestion in the *Guidelines* (p81), that there should be changes in the emphases on each of the three modes of calculating - mental, written and calculator.

As can be seen from Table 5.4, **ways of teaching** constituted the one of the five aspects of mathematics teaching which teachers felt had changed least with the implementation of the *Guidelines*. Nonetheless, substantial numbers of teachers offered suggestions as to how their teaching had changed to indicate that real change in teaching practices was taking place. This was happening despite the fact that the *Guidelines* did not have the stated purpose of making recommendations about methods of teaching. Teachers had, on the evidence of their responses to this section, drawn some inferences from the *Guidelines* which clearly had persuaded them that some changes in teaching methods were indicated.

The greatest number of teachers (21%) said that they were using more practical activities in mathematics. This was surprising to the researcher, as greater use of practical activities had not been mooted explicitly in the *Guidelines*. Teachers, however, clearly saw this

change as being necessary to implement not only the content but also the spirit of the *Guidelines*.

Reference was made by 14% of participants to the greater variety of organisational structures they were now using, whilst on a similar theme, 10% highlighted 'more group work' as a consequence of the implementation of the *Guidelines*.

Table 5.4 shows that 87% of the respondents reported changes in the area of **planning** and of these, almost half felt that the changes were major. Of those who answered question 5(c), the open question which referred to the ways teachers planned mathematics, by far the greatest number referred to changes resulting from the use of 5-14 ideas and terminology as a framework for planning. The use of outcomes, strands and levels was referred to by 38% of the respondents and there had clearly been many attempts to devise new planning formats using these. Twenty five percent felt that more time was now spent on planning within the new framework, but this did not appear to cause any measure of resentment. There was a feeling that progression was easier to plan for and identify using the *Guidelines* and that their use also encouraged a better balance across the outcomes.

Two teachers remarked on the fact that there was less reliance on textbooks at the planning stage - a perceived tendency which they seemed to welcome. A similar number commented on the need to select a variety of contexts at the planning stage.

**Assessment** procedures were identified by 86% of responding teachers as an area in which change had taken place and there was a range of views as to what form these changes had taken. The introduction of national tests was clearly a major change, identified as such by 20% of teachers. Slightly less than this said there was more formal assessment and a similar number felt that assessment now was usually linked to the 5-14 document. Many references were made to the 5-14 framework of outcomes, strands and levels and how these had guided assessment practices. A few responses also referred to

the 'new' assessment formats which their schools had produced using the 5-14 framework. Several responses pointed out that teachers were still relying on their own judgement based on informal, continuous assessment.

There was also a feeling that there was now more assessment of a variety of types - national tests, formal, informal, group, self and practical assessment. There also appeared to be an acceptance of the extra amount of work which was needed to implement and maintain the increased amount of assessment identified.

There had been more changes in **recording** procedures than in any other of the five areas investigated in the questionnaire. A clear majority (56%) of the teachers responding felt that the changes in recording had been major.

The main points found were that:

- this was the area identified by most teachers as having involved the greatest amount of change
- most teachers felt that more recording was now being done, resulting in more detailed and precise records
- there appear to have been efforts throughout the country to derive new formats for recording mathematics using the 5-14 terminology and framework
- most teachers seem to have found the 5-14 framework helpful in devising recording formats. There were frequent reports of teachers and schools trying to devise single formats which could be used for all three purposes of planning, assessing and recording.

Not unnaturally the comments received in this section reflected many of those in the previous section, with most of them referring to the use of new formats which applied the 5-14 framework of outcomes, strands and levels.

## Commentary

It was hardly surprising that the Problem Solving and Enquiry outcome was identified as the area of greatest change in content, since one of the goals of RDG 2, had been to augment and enrich children's mathematical experiences by creating a curriculum which looked beyond the acquisition of the normal body of knowledge consisting of the traditionally learned facts, skills and concepts of mathematics.

It was interesting to note that some areas which might have been expected to feature in the responses in the section on content did not appear at all. There was, for example, virtually no mention of the increased emphasis on mental work which had been advocated in the *Guidelines*, nor did anyone refer to the need to teach skills of approximation or estimation which should accompany a greater use of calculators. There were no unsolicited comments about the need to reconsider the whole balance between the amounts of time devoted to each of the three ways of calculating. This issue, however, will be explored in greater depth in Section 5.2.5. On the other hand, 14 people referred to the need for more practical work - an area which received very little prominence in the *Guidelines*, but which a number of teachers clearly felt was necessary for their proper implementation.

This view was echoed in the responses to the next section on ways of teaching, in which the responses given were of interest to the researcher since there was only one section of the *Guidelines* in which explicit reference was made to ways of teaching mathematics. This took the form of a brief reminder (SOED 1991, p57) of the four 'modes' first referred to by the Committee on Primary Education (1983). A second more indirect reference in the *Guidelines* to ways of teaching occurred in the Problem Solving and Enquiry outcome. Children were described as being involved in problem solving and enquiry when they were "adopting an investigative approach to learning concepts, facts and techniques" (SOED 1991, p48). This important aspect of the problem solving and enquiry outcome clearly has implications for the ways in which teachers should teach mathematics, but given the total absence of reference to it in the responses to this section,

it would appear to be an idea which teachers either had not yet taken on board, or were already using anyway, since it may have been recognised as a reiterated version of the 'enquiry' mode, one of the four referred to above.

The three areas of 'practical work', 'group work' and 'greater variety of organisations' which were mentioned most frequently by teachers in their responses to this section on ways of teaching, suggest that teachers realise that successful implementation of the *Guidelines* depends on a willingness to adapt their teaching approaches in these ways.

Planning was viewed as an area in which major changes had taken place, but teachers seemed to welcome these changes in a positive way. Many changes in planning practice had been agreed at staff meetings or following initiatives from school management teams, but all were consequent to the implementation of the *Guidelines*.

A feature of the responses to the section on assessment was the number of contrasting references to the more formal and structured approach represented by assessments such as the national tests, as opposed to a continued reliance on continuous, informal and observational processes. Although the impression gained from reading the range of comments in these responses is one of more perceived assessment, there seemed to be an acceptance among teachers of this situation with few, if any, participants expressing any reservations about the changes in assessment following on from the implementation of the *Guidelines*. Teachers' acceptance of the apparent increase in the assessment load may be related to the support provided by the more structured framework of the *Guidelines* in tracking pupils' progression in mathematics, which was mentioned in fourteen responses.

The new structure provided by the *Guidelines* had clearly instigated, in many schools, a successful search for a new recording format to match the structure. Various experiments were referred to which described formats, some of which sought to serve a dual purpose of planning and recording. Whilst many teachers felt on the one hand that they were now doing more recording, they also felt that they could provide more detailed and precise

records using the *Guidelines*. This may be the reason for the additional record keeping not being seen as a burden.

### 5.2.3 Support available for teachers in implementing the *Guidelines*

#### Summary

The questions in this section sought to identify the sources of help which had been available to teachers in the implementation of the *Guidelines* and to ascertain which of these sources had been most appreciated by teachers. Teachers were also asked to specify factors which had caused them difficulty in implementing the *Guidelines* and to state which further forms of support would have been appreciated. The main points noted were as follows:

- Teachers had enjoyed a wide range of formal and informal support mechanisms, all of which were described as 'useful' or 'very useful' by most recipients.
- The most highly valued forms of support were (a) opportunities for both formal and informal meetings with colleagues in school to discuss the *Guidelines*, (b) extra time to themselves for reading, planning and thinking, (c) advice from promoted staff in schools.
- A disappointingly small percentage of teachers surveyed seem to have had sight of publications produced by the SCCC to provide advice which was designed specifically to help teachers implement the mathematics *Guidelines*.
- In terms of extra support required and hindrances identified, there was a plea for more time for thinking, reading, planning, discussions with colleagues and for organising new materials and resources.
- Additional help with the Problem Solving and Enquiry outcome was the only content area referred to.
- There were requests for more resources, physical, financial and printed, with expressions of need for new textbooks and schemes which matched the content and structure of the *Guidelines*.
- Contrasts were noted between the views of Scottish teachers in this context and those of teachers in England and Wales who expressed different views about areas of

difficulty. Factors such as class size, classroom organisation and management, self-confidence in mathematics and familiarity with 'Handling Data', all of which featured prominently in a similar survey in England and Wales (SCAA, 1993), were not mentioned by teachers in this survey.

Detailed findings

Question 6 in the questionnaire sought to ascertain not only which types of support had been available, but also which of those given had been most appreciated. Table 5.5 gives a summary, in percentages, of the responses to question 6, which asked,

*"Please indicate which of the following forms of support you have had and how useful each has been. (Please tick the 'Yes' box and one other box for each form of support you have had)".*

Note that the numbers given in columns 3 to 8 of Table 5.5 are percentages of those respondents referred to in column 2, who said they had received each form of support.

**Table 5.5**  
*Support given to teachers and their reactions to it. (n=328)*

	Yes	very useful	quite useful	of limited use	not useful	useful col.3 + col.4	not useful col.5 + col.6
In-service courses delivered by Regional staff	60	18	53	27	2	71	29
In-service courses delivered by College of Education Staff	28	23	46	23	8	69	31
Advice from Regional advisers or other Regional personnel who have come to your school	41	28	48	20	4	76	24
Informal conversations with colleagues about the guidelines	92	63	32	5	0	95	5
Organised meetings with colleagues from your school	82	65	32	2	1	97	3
Extra time to yourself for reading, planning and thinking eg. P.A.T. time	77	73	25	2	0	98	2
Advice from your school's promoted staff	76	58	37	4	1	95	5
The staff development pack "Mathematics 5-14 Exemplification"	40	20	56	18	6	76	24
"5-14 Catalogue: Mathematics" (SCCC 1993b)	32	19	51	25	5	70	30
"5-14: A Practical Guide" (SOED 1994)	54	28	49	22	1	77	23
Printed material produced by your Region eg. 5-14 Regional Guidelines	57	37	50	12	1	87	13
Appropriate commercially published material	64	50	42	8	0	92	8

As may be seen from this table the two forms of support most commonly experienced were those which involved teachers meeting with colleagues. Informal conversations and organised meetings with colleagues enjoyed 95% and 97% ratings respectively in terms of the numbers who found them either 'very useful' or 'quite useful'. The other aspects of support which matched these in terms of perceived usefulness were 'extra time for yourself for reading, etc.', 'advice from your school's promoted staff' and, to a slightly lesser extent, 'appropriate commercially published material'. These received ratings of 98%, 95% and 92% respectively.



Just over a quarter of the participants in the survey had taken courses delivered by college of education staff and fewer than half claim to have seen the Scottish Consultative Council on the Curriculum (SCCC) publications, *Mathematics (5-14) Exemplification* (SCCC, 1993a) and *5-14 Catalogue: Mathematics* (SCCC, 1993b) Courses, advice and printed materials received from regional authorities all received usefulness ratings in excess of 70%.

A cross tabulation of the data from question 6, 'forms of support given to teachers' and question 2, 'teachers' views of the *Guidelines*', shown in Appendix 5.1 and in Table 5.6, reveals some additional information.

Table 5.6

*Data from a cross tabulation of forms of support given to teachers (question 6) with teachers' views of the Guidelines (question 2).*

	1	2	3	4	Differences	
	had and liked	had and disliked	had not and liked	had not and disliked	col.1-col.3 liked had-hadn't	col.2-col.4 disliked had-hadn't
In-service courses delivered by Regional staff	83.1	6.7	74.6	9.0	8.5	-2.3
In-service courses delivered by College of Education Staff	83.9	6.9	78.6	7.9	5.3	-1.0
Advice from Regional advisers or other Regional personnel who have come to your school	85.3	6.2	76.9	8.6	8.4	-2.4
Informal conversations with colleagues about the guidelines	81.0	7.6	55.6	11.2	25.4	-3.6
Organised meetings with colleagues from your school	80.0	7.9	78.5	7.9	1.5	0.0
Extra time to yourself for reading, planning and thinking eg. P.A.T. time	80.0	8.4	76.5	5.9	3.5	+2.5
Advice from your school's promoted staff	79.8	8.9	78.9	3.2	0.9	+5.7
The staff development pack "Mathematics 5-14 Exemplification"	86.0	3.1	75.8	11.0	10.2	-8.0
"5-14 Catalogue: Mathematics" (SCCC 1993b)	86.4	3.9	76.9	9.7	9.5	-5.8
"5-14: A Practical Guide" (SOED 1994)	78.8	6.5	81.4	9.7	-2.6	-3.2
Printed material produced by your Region eg. 5-14 Regional Guidelines	78.5	8.0	81.4	7.8	-2.9	+0.2
Appropriate commercially published material	82.3	7.2	76.2	8.6	6.1	-1.4

The item which stands out in this table is 'informal conversations with colleagues...'. There was a much greater difference in the views of the *Guidelines* between those who

had received this support and those who had not, than amongst any other group. Table 5.5 showed that this item was described as 'useful' by 95% of those receiving it. The cross tabulation (Appendix 5.1) showed that 81% of those who had had this support liked the *Guidelines* but that only 55.6% of those who had not experienced this support liked the *Guidelines*. This was about 20% lower than any other group who 'liked' the *Guidelines*. This suggests that this item, in addition to being one of the most appreciated forms of support received with a 95% approval rating (in Table 5.5), was also one of the most influential in shaping positive views on the *Guidelines*. This result should be treated with some caution however, since only 18 teachers had not experienced this form of support.

Another interesting item was *Mathematics 5-14 Exemplification* (SCCC, 1993a) which, according to the results in Table 5.5 had been experienced by only 40% of participants. It showed the second biggest difference in their liking of the *Guidelines* between those who had experienced this publication and those who had not. The *5-14 Catalogue; Mathematics* (SCCC, 1993b) showed a similar but slightly smaller difference between those who had seen it and those who had not. These two results, which suggest that the publications involved seemed to have been influential in changing teachers' views of the *Guidelines*, should be welcomed by the SCCC and should help to compensate for the disappointingly small proportion of teachers who actually saw these particular pieces of support material (40% and 32% respectively)

There were few responses to the request for information about 'other forms of support you have had' which concluded question 6. Some teachers had taken part in local authority working groups and this had helped them to 'get into' the *Guidelines*. Others had attended conferences and there were some references to extra resources having been made available in schools.

Questions 7 and 8 were open questions which invited participants in the survey to specify what forms of further support they felt they needed and which factors had caused them

greatest difficulty in implementing the *Guidelines*. There was a very close match to the responses to these two questions. The single most common issue, by far, was a plea for more time to adapt to the *Guidelines*.

In question 7, 24% expressed a need for more time. Time was needed for reading, planning, discussion with colleagues and for organising resources and teaching materials.

Lack of time was also cited in the responses to question 8 as the factor which caused the greatest difficulty in implementing the *Guidelines*. Forty four percent of the sample referred to the lack of time for the purposes mentioned in relation to question 7 above.

The second most commonly cited factor of difficulty and area of help needed was that of resource provision, with 20% of the sample referring to the need for more resources. In addition, 20% in question 8 said that the lack of resources caused difficulties for them in trying to implement the *Guidelines*.

Other support needed included help with the problem solving and enquiry outcome (8%), more money for resources (7%) and help with matching existing textbooks to the 5-14 framework (6%). These were also cited in the responses to question 8 as factors causing difficulties, with similar frequencies. In addition, 'too much change' and 'new terminology or jargon' also caused difficulties for teachers.

### Commentary

It is clear that for most Scottish primary teachers lack of time was seen as the greatest obstacle to successful implementation of the *Guidelines*, with lack of resources, both material and financial, also major factors. The only area of content mentioned as causing difficulty was that of problem solving and enquiry. This contrasts sharply with the results of a similar survey carried out with teachers in England and Wales, (SCAA, 1993) in which most of the difficulties highlighted were related to content issues. 'Using and Applying Mathematics' (Ma1) was identified by teachers as a major area of difficulty as

were topics such as 'handling data' and 'algebra' and teachers' ability to teach these. Other difficulties mentioned in the English study included difficulties in the classroom concerned with organisation, management and differentiation, and teachers' lack of experience or confidence in their own knowledge. There was also a feeling that there was "too much to get through" (Brown *et al.*, 1993, p5). The only common area of difficulty in the implementation of both the English National Curriculum and the *Guidelines* was that of 'Problem Solving and Enquiry', and its English equivalent, 'Using and Applying Mathematics', attainment target Ma1.

The results in this survey are similar to those observed by Harlen and Malcolm (1993) in another study related to the implementation of the *Guidelines*, in which 'time' was also the major area of difficulty specified by teachers. In that study there was also a request from teachers for more in-service courses, a request repeated by only 17 teachers in this survey. These views contrast sharply with those found by Bennett *et al.* (1992), who, in a survey of the National Curriculum in England and Wales, found that two thirds of the teachers sampled, when questioned about the quality of INSET received, considered it to have been either 'poor', 'very poor' or 'inadequate'. Campbell *et al.* (1991, p44) also reported that,

The most savage ire however, was reserved for time wasted in INSET training.  
.....most teachers regarded it with something approaching contempt.

As noted in Table 5.5, about 70% of Scottish primary teachers felt that in-service courses delivered by either regional or college of education staff had been either 'very useful' or 'quite useful'.

As noted previously, the numbers of teachers claiming to have seen the SCCC publications *Mathematics 5-14 Exemplification* (SCCC, 1993a) and *5-14 Catalogue: Mathematics* (SCCC, 1993b) were surprisingly and disappointingly small (40% and 32% respectively) considering that these were sent to every school in Scotland. This would appear to indicate that sending one copy of staff development materials to schools

is no guarantee that it will be seen by all the teachers. The fact that these two forms of support, as noted above, seem to have been effective in supporting a positive view of the *Guidelines* among teachers, suggests that more consideration needs to be given to ensuring that all printed materials designed for teachers should in fact be seen by them.

It is noteworthy in this survey that so few teachers mentioned class size as a factor causing difficulty in the implementation of the *Guidelines*. In question 7, three teachers referred to the need for smaller classes whilst seven responses to question 8 made a similar reference. Once again, these numbers contrast sharply with those in England and Wales. In a study by Wragg *et al.* (1989), 45% of teachers sampled mentioned class size as a constraint.

In general, the results of this section of the questionnaire show that whilst teachers were appreciative of the support which they had received for the implementation of the *Guidelines*, there was still a feeling that more follow-up support was needed in the previously specified areas relating to time, resources and problem solving help.

#### **5.2.4 Teachers' views on the Problem Solving and Enquiry outcome**

##### **Summary**

This section was included in the survey, since it had been identified at the pilot study stage as the aspect of content which provided most teachers with a new challenge. In this case teachers in Scotland had concerns similar to those of their colleagues in England and Wales, for whom the corresponding attainment target Ma1, 'Using and Applying Mathematics', had caused difficulties. Perhaps because this outcome was new to most primary teachers, there was a number of key issues emerging from the survey. These were:

- Most teachers agreed that the Problem Solving and Enquiry (P.S.E.) outcome was one of the most difficult aspects of the *Guidelines* for them.
- There was general agreement on the need for more help and guidance on aspects of the P.S.E. outcome in the following areas:

- attaching levels to problem solving activities in the absence of attainment targets and levels in the *Guidelines*
  - a need for examples of problem solving activities with levels attached
  - defining progression within a primary stage, or over the whole range of stages from P1 to P7
  - differentiating problem solving activities
  - assessing, recording and reporting children's problem-solving activities.
- These requests for help stemmed from the fact that the P.S.E. outcome was not defined in the same explicit terms of strands, targets and levels, as the other three outcomes. The 'pragmatic approach' to defining progression in this outcome, recommended on page 9 of the *Guidelines*, was seen as inadequate to meet most teachers' planning needs.
  - Teachers showed a good awareness of the need to develop a range of problem solving strategies and appropriate attitudes in their pupils.
  - A quarter of the teachers sampled were still unclear about the three aspects of the P.S.E. outcome as described on page 48 of the *Guidelines*.

### Detailed findings

Question 9 was designed to survey teachers' feelings about the P.S.E. outcome. Table 5.7 shows the responses to the request:

"Please tick the box which best matches your level of agreement with each of the following statements about the problem solving and enquiry (P.S.E.) outcome".

Table 5.7  
*Teachers' feelings about the PSE outcome. (n=328)*

	strongly agree	agree	disagree	strongly disagree	no response	agree col.2 + col.3	disagree col.4 + col.5
The P.S.E. outcome was one of the most difficult aspects of the Guidelines for me	23.8	50.6	20.5	2.1	3.0	74.4	22.6
I feel that I cope well with matching P.S.E. tasks to children	3.0	55.3	36.6	2.1	3.0	58.3	38.7
I would appreciate more guidance about progression in P.S.E.	30.5	49.2	15.5	2.4	2.4	79.7	17.9
My own background in maths makes me feel insecure about teaching P.S.E.	7.0	26.5	50.0	13.1	3.4	33.5	63.1
I am satisfied with what I am doing with my class in P.S.E.	4.6	48.5	39.8	3.4	3.7	53.1	43.2
I would have liked examples of P.S.E. with levels attached	38.1	50.0	6.7	1.2	4.0	88.1	7.9

Almost three quarters of the teachers sampled agreed that this outcome had been one of the most difficult aspects of the *Guidelines*. It is also clear from this table that issues relating to levels of attainment and progression within the outcome were posing particular difficulties for teachers, with 80% requesting more guidance about progression and 88% agreeing that they would have liked P.S.E. examples with levels attached. The suggestion that teachers' own mathematical background might contribute to feelings of insecurity about the outcome did not meet with general agreement, with 63% disagreeing.

A slight majority of teachers expressed satisfaction with their current classroom practice in P.S.E. and a similar number felt that they were coping well with matching P.S.E. tasks to children. In both cases, however, a substantial minority (43% and 39% respectively) felt no such satisfaction.



Question 10 sought to explore teachers' awareness and use of the three different aspects of the P.S.E. outcome as articulated on page 48 of the *Guidelines*. Whilst most teachers seemed to be clear about these three aspects, fewer than two-thirds of them said that their teaching programme included all of them. Amongst the three aspects, there was no one which was significantly more popular with teachers than the others (Tables 5.8 and 5.9). These aspects of problem solving, as described on page 48 of the *Guidelines*, were included in the instructions to question 10. Table 5.8 shows the percentages of teachers responding to the request:

- 1. *Adopting an investigative approach to learning concepts, facts and techniques*
- 2. *Working on tasks designed specifically to highlight the merits of certain approaches to mathematical thinking*
- 3. *Using their mathematics in an enquiry which could be part of a cross-curricular study*

*"The following questions refer to these three aspects of problem-solving and your views on them. Please tick the box which most nearly matches your own view."*

Table 5.8  
*Teachers' understanding of the three aspects of PSE. (n=328)*

	strongly agree	agree	disagree	strongly disagree	no response	agree col.2+col.3	disagree col.4+col.5
I am clear about the three aspects of P.S.E.	9.2	64.3	23.5	1.8	1.2	73.5	25.3
My teaching programme in P.S.E. includes all three aspects	4.3	58.8	34.8	0.9	1.2	63.1	35.7
The examples in the Guidelines (p.48) which illustrate these aspects are helpful	3.0	64.4	24.7	2.1	5.8	67.4	26.8

Table 5.9 shows the percentages of teachers responding to the question,

*"Which of the three aspects do you feel most comfortable with in your teaching? (Please tick the appropriate box(es))."*

Table 5.9  
*Teachers' feelings of comfort about the three aspects of PSE*

Aspect(s)	1	2	3	no response
Percent responses. n=328	42	38	36	4

Since some teachers chose more than one aspect in responding to question 10D, the total of the percentages in Table 5.9 exceeds 100%.

Question 11 returned to some of the issues raised in question 9 and in addition raised questions about assessment, recording and reporting in the P.S.E. outcome. The results, in percentages, are given in Table 5.10 below.

*"At what stage are you, with respect to each of the following P.S.E. activities? (Tick one box to show what stage you are at and indicate with a tick if you would appreciate advice)."*

Table 5.10  
*Stages reached in implementing the PSE outcome. (n=328)*

	not yet started	started but not far on	reasonably well on	no response	would appreciate some advice
Developing a structured programme in P.S.E. for the year	17.1	39.3	37.8	5.8	32.6
Matching P.S.E. activities to 5-14 levels	10.1	37.2	46.3	6.4	29.9
Differentiating the P.S.E. activities used in my class	14.3	41.5	39.3	4.9	30.8
Developing a procedure for assessing my pupils' P.S.E. work	31.1	39.3	20.1	9.5	42.4
Producing a recording and reporting format for each pupil's P.S.E. attainments	43.6	32.3	14.0	10.1	41.5

The evidence in Table 5.10 suggests that most teachers were currently engaged in the first three activities listed, with between 10% and 20% not yet started. The picture in relation to teachers' work on assessing, recording and reporting children's work in the P.S.E. outcome was somewhat different and suggested that teachers had not yet established systems in these areas. This is shown by the numbers who had 'not yet started' to develop assessment procedures or recording/reporting formats (31% and 44% respectively) and the numbers who 'would appreciate advice' on these topics (42% in each case). A cross tabulation of each part of question 11, the results of which are shown in Table 5.10, was done with question 36, about the stages taught by the responding

teachers. The responses given to question 11 were spread evenly across the stages with no differences noticeable between teachers of different stages.

Question 12 in this section of the questionnaire referred to the 'Starting-Doing-Reporting' cycle of problem solving as described in the *Guidelines* (p12). In particular, information was being sought on how teachers were addressing the need to develop their pupils' skills in 'reporting' their problem solving activities. Table 5.11 shows the results in percentages.

*"The Guidelines, (p12), describe 3 steps in the problem solving process as Starting, Doing and Reporting. This question concerns what your pupils do in the Reporting stage. In each case please tick the box which best describes what your pupils do."*

Table 5.11  
*How pupils report on their PSE activities. (n=328)*

<i>They report on each problem they have solved by:</i>	always	often	occasionally	never	no response
Keeping a record of their solutions in their jotters or folders	25.6	30.5	30.2	10.7	3.0
Explaining to their group (or class) how they did it	4.3	41.8	50.9	1.8	1.2
Explaining to the teacher how they did it	20.1	57.0	21.1	0.3	1.5
Referring orally to the strategy (or strategies) used	11.8	49.4	28.4	7.0	3.4
Writing down the strategy (or strategies) used	4.9	19.5	46.6	24.7	4.3
Displaying their solutions	9.8	36.3	47.8	3.7	2.4

All of the statements in Table 5.11 represented, in the researcher's view, good practice in children's problem solving. An analysis of the responses to questions 9 and 12 shows a high correlation between the responses to question 9B and the responses to all parts of question 12, showing that those who felt in question 9B that they were coping well with matching PSE tasks to children were those who were encouraging reporting. A similar relationship, in virtually all cases at the 99% confidence level (Appendix 5.2), existed

between those who were satisfied with what they were doing in PSE (Q9E) and all items in question 12.

The next two questions asked about the amount of time pupils were spending on problem solving activities and how their teachers felt about these amounts of time. Teachers indicated that 57% of pupils were spending between 30 minutes and one hour per week on P.S.E. and 41% were spending less than 30 minutes. A cross tabulation of these two questions shown in Table 5.12 shows that two thirds of teachers surveyed felt that the amount of time spent on P.S.E. was 'about right' and one third felt they spent too little time on it. Of the one third who felt they were spending too little time on it, two thirds spent less than thirty minutes and one third spent between thirty minutes and an hour on problem solving each week.

Table 5.12  
*Cross tabulation of question 13 with question 14*

Time spent by pupils each week	Teachers' views about time spent on PSE			row totals
	too little	about right	too much	
less than 30 minutes	67	58	2	127 40.3%
between 30 & 60 minutes	33	143	4	180 57.1%
more than 60 minutes	1	7	0	8 7.5%
column totals	101 32.1%	208 66.0%	6 1.9%	315 100%

The reasons given by teachers for spending too little time on P.S.E. were predominantly to do with lack of time to fit it in to what was seen as an already heavy mathematics curriculum. Other reasons given were, 'not enough material' and 'lack of familiarity or confidence' with the outcome. Those teachers who felt that their time allocation was 'about right' said that there was no time to do more and that they felt they had achieved a good balance across the four outcomes in mathematics.

Question 15 in the questionnaire was an open one which asked,

*"Which P.S.E. skills, strategies or attitudes do you hope will have been acquired by the end of this academic year by (a) your highest attaining pupils and (b) your lowest attaining pupils?"*

The responses given to parts (a) and (b) are summarised below under the two headings 'skills and attitudes' and 'strategies', with the frequency of each response given.

highest attainers		lowest attainers	
<u>skills and attitudes</u>	f	<u>skills and attitudes</u>	f
get started	37	be able to start	52
report in a variety of ways	37	have confidence	40
work independently	31	be able to report their answer	28
have confidence	30	come to a conclusion	23
record their work	26	use a variety of strategies	8
come to conclusions	21	work independently	8
check their solutions	17	be able to cooperate	5
adopt an investigative approach	9	be interested	5
choose an appropriate strategy	8		
<u>strategies</u>		<u>strategies</u>	
look for a pattern	37	draw a picture/diagram	21
guess, check and improve	32	guess, check and improve	21
make a list/table	29	look for a pattern	15
reason logically	28	try a simpler case	7
draw a picture/diagram	26	act out the situation	7
act out the situation	21	make a list	6
try a simpler case	21	reason logically	4
work backwards	18	work backwards	3
all strategies	13	make a model	2
make a model	2		

## Commentary

There is little doubt that the P.S.E. outcome is the one which has proved difficult for many teachers to implement. As well as the 74% of participants who expressed this view in this survey, similar numbers did so with respect to the corresponding section of the National Curriculum Mathematics in England and Wales, the Using and Applying Mathematics attainment target, Ma1;

Teachers are finding the process of using and applying mathematics difficult to integrate into their work. (NCC, 1990, p15)

This finding was endorsed by one of the recommendations in the *Evaluation of the implementation of National Curriculum mathematics at key Stages 1, 2 and 3*:

Award priority to 'Using and Applying Mathematics' (Ma1) for the provision of support to teachers. Support should include extensive extended INSET, further exemplar materials, advice to publishers and stimulus for in-school reflection and development. (SCAA, 1993, p16)

Most of the questions in this section of the questionnaire tried to probe into the reasons for teachers' difficulties with the outcome.

Many of these difficulties resulted from the fact that no levels were defined or described within the P.S.E. outcome. This meant that whilst teachers felt reasonably comfortable in working within the defined 5-14 levels in the other outcomes, the lack of guidance given in P.S.E. meant that they had no such feelings of confidence in it. This was illustrated by the facts that 39% of those surveyed felt they did not cope well with matching the P.S.E. tasks to their pupils and 88% of the sample would have liked examples of problem solving tasks with levels attached. This absence of levels within the outcome creates the associated difficulty of defining progression in problem solving and once again a large majority (80%) of teachers said they would appreciate more guidance about progression in P.S.E.

This figure is not surprising since no help or guidance on progression in problem solving skills or strategies is given in the *Guidelines*. Indeed it was explicitly stated that:

At present there is insufficient research evidence or practical experience to define progression precisely and a pragmatic approach is recommended. (p9).

One manifestation of a 'pragmatic approach' has been that individual teachers have begun to implement a problem solving programme within their own classes with little knowledge or regard for what problem solving experiences have gone before or will come after their particular stage. They have been helped in these early attempts by a number of commercially produced sources of problem solving tasks which somewhat arbitrarily attach levels to particular problems. However unreliable such a labelling of problems may be, it has nonetheless been appreciated by teachers as providing them with a starting point for developing a problem solving programme for their classes.

This state of affairs in schools appears to be the cause of a common observation by members of HM Inspectorate in their reports on school inspections. Phrases such as 'lack of progression' and 'lack of structure' are commonly used to describe schools' problem solving courses. This suggests that something more than the *Guidelines'* 'pragmatic approach' is now being expected from schools by the Inspectorate.

It appears unlikely that teachers' requests for advice and guidance in defining progression in problem solving will be met in the foreseeable future without a concerted and perhaps nationally instigated and funded staff development initiative. This idea has already been mooted in England and Wales in a recommendation to:

Consider commissioning work to study and report on successful collaborative whole-school planning relating to 'Using and Applying Mathematics' (MA1). (SCAA, 1993, p16)

In addition to the evidence in this survey, anecdotal evidence suggests that help with this aspect of the implementation of the P.S.E. outcome is being sought with increasing

degrees of urgency by schools which do not have the expertise or experience to fulfil this task themselves.

Any such staff development initiative would do well to address the issue of defining the problem-solving outcome more explicitly, since more than one quarter of teachers surveyed remained, five years after the publication of the *Guidelines*, unclear about the three important aspects of the P.S.E. outcome. This may be related to the fact that this interpretation of the meaning of the term 'problem solving' does not appear in the *Guidelines* until page 48. Only 63% of the sample claim to include all three aspects in their P.S.E. teaching programmes, although illustrative examples are given.

In the absence of a set of clearly delineated attainment targets in the P.S.E. outcome, many teachers see the strategies described in the *Guidelines* as targets. Of the teachers sampled, 89% claim to have their pupils refer orally to the strategies they used in problem solving and the responses to question 15 suggest that teachers see a clear need to impart a knowledge of strategies to pupils of all attainment levels. This is to be welcomed as knowledge and awareness of a variety of strategies is considered, by virtually all commentators on problem solving, to be an essential component in the range of skills which are generally recognised as constituting an expertise in problem solving. Although there seems to be a consensus that knowledge of a range of problem solving strategies is desirable, opinion is still divided about how these strategies should be taught or learned and conflicting messages are being given to teachers. Some commentators, for example, caution against an explicit 'teaching' of strategies. Lester (1994, p666) remarks that:

Teaching students about problem solving strategies ..... does little to improve their ability to solve mathematics problems in general.

Stacey and Groves (1988, p205) assert that:

We do not endorse the teaching of extensive, perhaps even classified lists of strategies.



Schoenfeld (1988, pp11-12) makes a similar point in a number of ways;

There is much more to thinking mathematically than knowing a few problem solving strategies.

and later, on the same page,

In much curriculum development we see adjunct units on problem solving strategies, a week on this problem solving technique or that (eg. 'Here's how to find patterns'), taught and tested as a separate bit of subject matter. This kind of approach trivialises mathematical thinking. It may be better than nothing at all, but not by much.

Peterson, Carpenter and Fennema (1989) argue that pupils' problem solving achievement was significantly related to their teachers' knowledge of the pupils' problem solving knowledge. This teachers' knowledge of their pupils' problem solving knowledge was, in turn, significantly related to the teachers' questioning pupils about the processes they used to solve problems. They also claimed that there was a significant negative correlation between pupils' problem solving achievement and teachers' explaining the problem solving process.

By contrast there are, in the literature, a number of references to the efficacy of teaching strategies. Burkhardt *et al.* (1986, p214) state categorically that:

The teaching of strategies is effective; it also provides teachers with some place for their explanatory skills. Further, it is generally safe to assume that if a strategy or technique is not taught explicitly, students will not learn it.

Stacey and Groves (1988, p205) also agree that pupils should have strategies made explicit:

Simple problem solving strategies, which arise naturally in a variety of contexts and whose use can clearly be seen to enhance the process, should be made explicit to students.

They do not, however, advocate direct teaching of lists of strategies, commenting instead that:

we see value in drawing out of solutions from carefully chosen problems a small selection of apparently simple strategies

These latter two approaches agree with Holton *et al.*'s (1996) 'Heuristics first' approach, whereas the earlier comments seem to match their 'Problem first' model. As the teachers in this survey seem to appreciate, pupils do need to grow into an awareness of problem solving strategies. Whilst it is beyond the scope of this survey to comment on how best this might be achieved, there is still an obvious need to offer teachers advice on the most effective way of helping pupils to acquire a knowledge of strategies.

The responses to question 15 demonstrated that teachers did not view the teaching of problem solving as solely the imparting of knowledge about strategies. There were frequent references to the need for positive attitudes and an awareness of skills which, whilst by no means unique to P.S.E., still assume a greater importance in this context. The abilities to 'get started', 'report solutions' and to 'have confidence' were referred to with sufficient frequency to suggest that teachers now have a good breadth of understanding of the P.S.E. outcome. The importance of developing positive attitudes through the establishment of a problem solving environment has been well documented by researchers such as Burton (1986).

### **5.2.5 Teachers' views on the specific issue of calculating**

#### **Summary**

This section in the questionnaire sought to determine the amount of change which had taken place in teachers' use of the three modes of calculating and to assess the extent to which teachers had taken on board some significant ideas from the *Guidelines* which related not to the content taught, but rather to the approaches used in the general area of calculating. This was of particular interest since the issue of calculating did not relate directly to outcomes, strands or targets and as such the messages contained in it may have had less impact. The researcher was particularly interested in measuring the extent to

which teachers had responded to what has become a well known paragraph in the *Guidelines*,

This suggests some changes in emphasis in teaching computation: mental calculation will require more attention, written methods need not always be standardised on a concise but difficult to understand algorithm ..... and new skills in using a calculator will need to be learned. (SOED, 1991, p81)

The main findings were:

- Before the implementation of the *Guidelines*, teachers estimated that of the mathematical calculations which were carried out in their classes, most involved written work, fewer than a quarter were done mentally and less than 10% used calculators.
- The corresponding figures after the implementation show that some change of emphasis had taken place, especially with respect to the use of calculators. Many teachers expressed the view that still more emphasis should be given to the place of mental algorithms for calculating.
- The use of calculators appeared to have doubled, and there is evidence that most teachers were seeking to develop children's personal non-standard mental and written algorithms in line with the advice given in the *Guidelines*.
- Half of the sample claimed to have increased the practice of estimating answers before calculating and of using calculators to check answers arrived at by other means.

### Detailed findings

The first question asked teachers to estimate the relative proportions of the total time spent by their class on calculating, i.e. what proportions were devoted to each of the mental, written and calculator modes. Before this, they had been reminded that 'mental calculation' did not include recall of number facts. Table 5.13 shows the average of the percentages given **before** and **after** the implementation of the *Guidelines* with the amount of change in each of the three modes expressed as a percentage of pre- 5-14 practice. The percentages given in this table are of the 256 teachers who responded to the request:

*"Estimate the percentage of the total time spent on calculating allocated to each method."*

Table 5.13  
*Changes in the modes of calculating. (n=256)*

	Pre 5-14 practice	Practice after 5-14 implementation	Percentage increase(+) or decrease (-)
written	69.7	60.4	-15.3
calculator	8.1	17.4	+114.8
mental	21.4	24.1	+12.6

Teachers were then asked to comment on the proportions which they had given, the averages of which appear in Table 5.13. Of the 256 responses, 71% felt that the proportions were 'about right' and 29% felt that they were 'in need of review'. Those who indicated 'in need of review' were asked to specify how the proportions should be changed. Half felt more time was needed on mental calculation, 26% felt similarly for calculators and 10% wanted less time on written methods.

Participants were next asked to estimate the percentages of calculations done in each of the three ways by themselves in real life, if possible excluding their professional activities as a teacher. The results shown in Table 5.14, differ markedly from those in Table 5.13.

Table 5.14  
*Modes of calculating used by teachers themselves. (n=265)*

	%
written	25.9
calculator	15.4
mental	58.7

Teachers were left to draw their own inferences from these differences which showed that mental calculating skills were used in real life much more than the other two modes. Whether or not teachers are aware of this, it is obvious from these results that classroom practice has not changed to reflect the need for greater attention to the development of

mental calculating skills and algorithms, which are referred to in the *Guidelines* as, "often more powerful" (SOED 1991, p81) than written algorithms. The *Guidelines* also recommend that "written methods of calculating should develop from mental ones." (SOED, 1991, p83)

Questions 19, 20, and 21 investigated teachers' use of personal, non-standard methods of carrying out mental and written calculations. Eighty seven percent of teachers claimed that they did encourage their pupils to use personal mental methods and of those, 75% did this for all pupils, although 16% did not do so with their lowest attainers. Question 20 asked participants about the origins of their own mental methods of carrying out the three calculations  $120 - 87$ ,  $99 \times 6$  and  $78 + 13$ .

Table 5.15 shows that approximately one half of the sample think that the method they used was one which had been taught to them in school. This seems to the researcher to represent a disappointing indictment of mathematics education, in that such a large proportion of those asked, claim to rely on processes or algorithms which seem likely to be self-taught. These figures would once again seem to endorse the statement on page 81 of the *Guidelines* and referred to earlier, on the need to reconsider the emphasis given to each of the three modes of calculating. Table 5.15 shows the percentages responding in each way. The question asked:

*"Think about the mental method you would use to perform each of these calculations and tick the appropriate box to show how you learned it."*

Table 5.15  
*Origins of teachers' own mental methods. (n=328)*

	120 - 87	99 x 6	78 + 13
I was taught it at school	42.7	48.8	44.5
I worked it out for myself	40.5	34.5	34.7
Don't know	10.1	9.1	9.8
No response	6.7	7.6	11.0

With reference to personal, non-standard methods of written calculations, 58% of the sample said that they did encourage their pupils to use them and 38% said they did not do so. When these 38% were asked to give reasons for not encouraging non-standard written methods the reasons given (with frequencies in brackets) were as follows:

- The time was needed to teach standard algorithms (27)
- They did not agree with it (21)
- It was not applicable to infants (13)
- They caused confusion (13)
- There was no need for it (6)

Of those who did encourage their pupils to use personal written algorithms, most did it with all pupils, although a small number (8%) excluded the lowest attainers and a similar number restricted this practice to their highest attaining pupils.

The final question in this section of the questionnaire addressed the usage of calculators both pre- and post-5-14 implementation. As can be seen from the percentages given in Table 5.16, a significant increase, not only in the use of calculators, but also in calculator-related activities, had either taken place or was being envisaged as a result of the implementation of the Guidelines.

*"This question seeks to identify changes in your classroom use of calculators as a result of the implementation of the guidelines (Please tick two boxes for each part (a) - (d)."*

Table 5.16  
The use of calculators. (n=328)

How frequently do pupils in your class:	pre 5-14 practice					practice/likely practice after 5-14			
	not at all	very rarely	sometimes	frequently	no response	increase	decrease	about the same	no response
Use calculators?	10.4	28.4	43.6	10.1	7.5	64.3	0.9	22.6	12.2
Estimate answers before using calculators?	9.8	22.8	42.1	16.2	9.1	48.5	1.5	35.4	14.6
Use calculators to check answers arrived at by written means?	13.4	28.7	40.5	7.3	10.1	52.2	0.9	32.9	14.0
Check answers produced on a calculator by doing a second calculation?	22.3	29.0	32.0	7.0	9.7	39.3	3.0	41.5	16.2

This question (22) sought to explore teachers' usage of calculators and the extent to which this would be affected by the implementation of the *Guidelines*. A cross tabulation of the responses to question 22A1 with those from question 36, which covered the stages taught by teachers, shows that, of the numbers who answered both of those questions, 47% used calculators 'not at all' or 'very rarely' before the implementation of the *Guidelines*. This 47% was composed of 48 or 69% of the P1-3 teachers and 44 or 35% of the P4-7 teachers responding. This result was in line with what could be expected as it is more likely that teachers of younger classes would use calculators much less than those of older pupils, especially as the *Guidelines* made no mention of the calculator until level B which many children would not be starting until P3. A cross tabulation of questions 36 and 22A2, which asked teachers about their intended use of calculators after the implementation of the *Guidelines*, showed that 70% of teachers over all stages planned an increase in their use of calculators. Somewhat surprisingly, in the researcher's view, 61% of teachers at the P1-P3 stages indicated an intention to increase calculator

usage, as did 75% at the P4-P7 stages. The two cross tabulations referred to appear in Appendix 5.1

### Commentary

There was a strongly held belief in RDG2, the group responsible for the *Guidelines*, that, in reality, most people, if and when they had to do a calculation would, as a first choice, do it mentally if they could. If they could not, it was becoming likely that they would use a calculator. These views were shared by, among others, Shuard (1986), Fitzgerald (1988) and the authors of the National Curriculum Council's *Non-Statutory Guidance* (NCC, 1989, p.E2) who wrote:

In practice, both children and adults employ a mixture of techniques when doing calculations,

and,

This central place of mental methods should be reflected in an approach that encourages pupils to look to these (mental) methods as a **first resort** when a calculation is needed.

The results of question 16 (Table 5.13) showed that, although teachers seemed to have accepted the need to reconsider the relative emphasis given to each of the three modes of calculating, there still remained a heavy dependence on written methods. Of the 23% of teachers who felt that still more readjustment was necessary, almost half (49%) felt that more emphasis on mental calculations was needed. This, in the researcher's view, still leaves an unhealthy bias towards written methods and there remains a staff development need to persuade teachers to concentrate more on the development of children's mental algorithms. This point has subsequently been picked up and reinforced in the HMI report *Improving Mathematics Education 5-14*, (SOEID, 1997, p15) in which teachers are exhorted to, "follow the advice in 5-14 Guidelines to increase the attention paid to mental calculation...".

It was interesting to note that, whilst children in school spend more than twice as much time doing written algorithms than they do mental ones, yet teachers still recognise that



they, like most adults, will perform more than twice as many mental calculations as written ones. It is also clear, from the response to question 20, that many teachers are using mental processes they either taught themselves or are unaware of how and where they learned them. This can be interpreted in several ways. For example it could be argued that people's mathematical educational experiences could be commended for providing them with the necessary background conceptual structures to be able subsequently to devise their own personal mental algorithms. On the other hand, the system could be criticised for leaving them in a position where functional and pragmatic processes had to be self-taught and devised. Perhaps a raising of teachers' awareness of the power and usefulness of personal mental algorithms will persuade them of the need to help their pupils to develop their own in the supported environment of the classroom.

It was gratifying to note that the majority of teachers taking part in this survey claimed to be encouraging the use of personal, non-standard written algorithms. This may be in response to the statement in the *Guidelines* (SOEID, 1991, p81) that,

Written methods need not always be standardised on a concise but difficult to understand algorithm.

The *Non-Statutory Guidance* (NCC, 1989, p.E4) goes even further in this direction by asserting that,

It is proficiency in (traditional pencil and paper methods) which has in the past often been used as a measure of attainment in mathematics. Such a view can no longer be sustained and indeed excessive practise (sic) of traditional pencil and paper methods out of context, will act in an inhibiting way to the overall aim of raising standards in mathematics.

The move away from a total dependence on standardised algorithms, however suspect it may be in the present political climate, has been well established and justified in the work of people such as Hart (1981), Fitzgerald (1988), Sewell (1991) and Thompson (1993) who contend that people tend not to use teacher taught algorithms in the real world when they have a choice as to which method of calculation to use.

The responses to the final question in this section indicated that the majority of teachers at all stages of the school have already increased or envisage increasing the use of calculators, suggesting that much more use of the calculator would shortly be happening at all primary stages. This trend, however, now looks as if it will be halted with the publication of *Improving Mathematics Education 5-14* (SOEID, 1997, p9) with its recommendation that,

(the 5-14) Guidelines ....should be reviewed to delay the introduction of the calculator until the late primary or early secondary stages.

This advice seems to fly in the face of most recent research findings (Askew and Wiliam, 1995; Shuard *et al*, 1991) that access to calculators does not lead to a lowering of standards of numeracy.

#### **5.2.6 Use of microcomputers in school mathematics**

##### **Summary**

The availability, frequency of use and the purposes of computer usage were explored in this section. The key findings are:

- Most teachers had readily available access to between one and two computers for their classes.
- Children typically spent less than 15 minutes per week and in many cases less than half of that time working on mathematical activities on a computer.
- Most teachers felt that they should be making greater use of computers than they currently were, but were constrained from doing so by a number of factors.
- A need for computer-related staff development was expressed by many teachers. Help was needed to familiarise teachers with new types of software and applications and advice was sought on organising the use of what were seen as inadequate numbers of machines for the benefits of all pupils.

Detailed findings

Tables 5.17 and 5.18 show the percentages of teachers in the sample having access to the different numbers of computers and the lengths of time spent by pupils using them for mathematical activities.

*"How many microcomputers do you have readily available for use with your class. (Please tick the appropriate box)"*

Table 5.17  
*Availability of computers (n=328)*

Number of computers	0	1	2	more than 2	no response
% of teachers	5	55	38	2	0

*"On average, how long would each pupil, in the space of four weeks, spend working on mathematics on a micro, either individually or in a small group. (Please tick the appropriate box)."*

Tables 5.18  
*Mathematics time spent on computers (n=328)*

Time spent on the computer	Less than 30 minutes	30-60 minutes	1-3 hours	3-5 hours	more than 5 hours	no response
% of teachers	50.2	31.7	10.8	1.5	0.6	5.2

Table 5.19 shows the usage, in percentages, of various types of mathematical software before and after the implementation of the *Guidelines*. It is noticeable that software which is designed to support specific areas of learning, such as games, investigations or consolidation and practice programs, is the type mostly widely used throughout the school. The other types of software mentioned were generally used only in the upper stages. This accounts for the large numbers who say that they use them either 'very rarely' or 'not at all'. In all cases the actual or projected use after the *Guidelines* shows a marked increase.

*"This question seeks to identify ways in which your use of software has changed as a result of the implementation of the guidelines. For each of the following kinds of software, please indicate both how often you used it before 5-14 was introduced and how often you will now use it after the implementation of the guidelines."*

Table 5.19  
Types of software used. (n=328)

	pre 5-14 practice					practice/likely practice after 5-14			
	not at all	very rarely	sometimes	frequently	no response	increase	decrease	about the same	no response
Turtle graphics packages e.g. logo	44.5	17.4	23.5	4.0	10.6	42.9	3.4	37.8	15.9
Software designed to support specific areas of learning e.g. games, investigations, consolidation and practice programs	9.8	18.6	41.1	19.8	10.7	42.7	3.7	40.2	13.4
Graph drawing packages	42.1	19.5	25.0	1.5	11.9	46.3	2.5	33.8	17.4
Databases	44.2	21.6	20.4	1.3	12.5	49.4	1.5	32.3	16.8
Spreadsheets	51.8	17.1	15.9	0.6	14.6	41.8	0.9	39.0	18.3

When asked whether they felt that they should make greater use of computers in mathematics lessons than at present, 84% of the sample replied in the affirmative. The reasons offered for not doing so are in Table 5.20. Since many teachers gave more than one reason, the percentages given here total more than 100.

*"Please tick those reasons why you do not use micros as much as you think you should."*

Table 5.20  
Reasons given for not making greater use of computers (n=328)

There are too few micros available	45.1
I am not sufficiently familiar with the software which is available to me	39.3
It is too difficult to organise the use of one machine with a whole class	37.8
I feel that I need more in-service training to work well with micros	30.2
I find it difficult to fit the available software into the context of ordinary maths lessons	25.9
Not enough appropriate software	10.4
Not enough time	6.4

## Commentary

The availability of computers in primary classrooms and the length of time spent on them by pupils, as shown in Tables 5.17 and 5.18, tend to confirm the findings of Askew and Wiliam (1995) and Sutton (1991), that access to computers is not always equitable, with some pupils having much less than others.

Although a majority of participants had not used databases and spreadsheets before 5-14, the size of this majority (70%) at both the P6 and P7 stages was greater than might have been expected. Given the large proportion of teachers who had not used databases before 5-14, it is not perhaps surprising that there was a correspondingly substantial number signalling an increase in their use post - 5-14, as had been advocated in the *Guidelines*. This proposed increase was indicated by a majority of teachers at the P4-P7 range and by a minority at the P1-P3 stages, as shown in Appendix 5.1. Similar figures held true for the actual and intended use of spreadsheets.

This survey did not detect the same degree of uncertainty about databases as was noted by Wragg *et al.* (1989) who found that the corresponding National Curriculum attainment target caused a high degree of uncertainty amongst two thirds of the teachers surveyed in England. Other recent research in England (Askew and Wiliam, 1995) suggests that there has been a swing away from the more generic, content-free type of software represented by databases and spreadsheets, towards more specifically educational software designed for classroom subject use. Although this survey did not address this point explicitly, no evidence has surfaced to suggest that such a move has happened in Scotland. This may be related to the fact that currently used examples of databases and spreadsheets, which according to this survey are increasing in use, have been designed explicitly for classroom use, as opposed to earlier versions which may not have had such obvious classroom relevance.

There were two further salient features of the results of this section of the survey. The first was the effects of the implementation of the *Guidelines* on the uses of the computer

in mathematics. Between 40% and 50% of the sample indicated an increase in the use of all kinds of software mentioned. There was also a high correlation, at the 99% confidence level, between those teachers intending to increase their use of the various computer applications mentioned, with those planning an increase in their use of calculators.

The second was the feeling of most teachers that they should be making greater use of the computer than they do at present. The results in Table 5.20 indicate not only a perceived need for additional computers and appropriate software but also a need for additional staff development time. Staff development is requested for familiarisation with existing software, developing organisational structures to maximise the uses and benefits of an insufficient number of computers in most classes, integrating existing software into mathematics lessons and in general more in-service training in the use of micros. These results also confirm the findings of Williams *et al.* (1998) who identified lack of availability as the main inhibiting factor in computer usage among primary teachers. They, however, referred to computer applications rather than the availability of computers themselves. In their survey more than 10% of primary teachers gave lack of familiarity or skills as the main reason for not making greater use of databases and spreadsheets. No more than 3% expressed any inhibitions to their use of educational software packages. These figures, coming as they do several years after this survey was completed, tend to confirm the figures given in Table 5.19 where 60% of the sample claimed to use educational software prior to the introduction of the *Guidelines* and 42% of the sample indicated an intention to increase their use of them after implementation.

### **5.2.7 Teachers' views on the use of context in mathematics**

#### **Summary**

One of the principal recommendations of the *Guidelines* was that pupils should meet mathematics in a variety of contexts, the better to be able to appreciate the nature and the purpose of the subject and to be aware of its relevance and usefulness in situations directly affecting their lives. The main points arising in this section were:

- There was widespread acceptance of the context message among teachers, with virtually all respondents agreeing in theory with the recommendations of the *Guidelines*.
- Although most supported a greater use of context, many found the idea difficult to implement in practice.
- Given the list of sample contexts mentioned in the *Guidelines*, between one third and a half of the sample anticipated making greater use of each of these kinds of contexts as a result of the implementation of the *Guidelines*.
- The only context given for which teachers envisaged a decrease in use was in the reliance on textbook series. Although this was not explicitly signalled in the *Guidelines*, teachers seem to be accepting the spirit of the 5-14 message in lessening their dependence on 'the scheme'.

### Detailed findings

Ninety five percent of teachers were in agreement with the statement given in the *Guidelines* (SOED, 1991, p44) that,

Teachers need to identify or create contexts which support what is to be learned and which motivate their pupils.

Of the 95% of teachers agreeing with this, 66% had reservations about the statement, with 28% saying they found it 'difficult in practice'. The other reservations of the 42% of teachers who 'agreed but with reservations' were not explored in the survey.

Question 28 tried to ascertain whether the implementation of the *Guidelines* had effected change in the use of context in mathematics. The seven examples of contexts given were taken directly from the *Guidelines*. The results are shown in Table 5.21.

*"This question seeks to identify changes in your use of context as a result of the implementation of the guidelines. (Please tick two boxes for each part (a)-(g))."*

Table 5.21  
*Changes in teachers' use of context. (n=328)*

	pre 5-14 practice					practice/likely practice after 5-14			
	not at all	very rarely	sometimes	frequently	no response	increase	decrease	about the same	no response
Managing everyday situations	1.8	10.1	51.8	23.2	13.1	33.3	0.9	50.9	14.9
Designing and Making	2.1	23.2	49.4	12.8	12.5	49.7	1.3	34.7	14.3
Studying aspects of the environment	1.5	14.8	53.4	18.7	11.6	38.2	0.9	46.3	14.6
Investigating areas of science	3.4	20.1	53.0	11.3	12.2	45.1	1.3	39.9	13.7
Investigating areas of mathematics	1.6	13.7	57.3	15.5	11.9	46.4	1.2	39.3	13.1
Play, games and puzzles	0.3	6.4	48.2	34.1	18.0	32.0	2.1	54.6	11.3
Text book series	2.1	0.0	22.6	57.3	18.0	12.8	14.7	56.7	15.8

### Commentary

Most of the items in Table 5.21 show a similar pattern, with most teachers using all of these contexts to some degree before the implementation of the *Guidelines*. This, in the researcher's opinion, is a reflection of teachers' professional instincts to exploit a variety of appropriate learning opportunities whenever possible. In this instance the *Guidelines* appear to be endorsing existing good practice. The fact that teachers found the 'context' message an attractive one is evidenced by the numbers who indicated an actual or proposed increase in their use of such contexts. The 28% who found these ideas difficult to implement in practice do not seem to have allowed this fact to deter them from trying to do so.

One item does stand out from the rest in Table 5.21. This is the last one in the table (Q28G), referring to the use of textbooks. Not unexpectedly, the majority of teachers at all stages used textbooks frequently prior to 5-14. When cross tabulated with the stages taught by teachers (Q36), the data from question 28G show that slightly more teachers plan a decrease than an increase in textbook usage at virtually all stages (Appendix 5.1). Twenty one percent of those who responded to both of these items intended to decrease



their reliance on textbooks after the implementation of the *Guidelines*. Whilst it is gratifying to see that this number of teachers were prepared to take this step, which, although not explicitly recommended in the *Guidelines*, is nonetheless in keeping with their spirit, these figures must be considered alongside the fifteen percent who plan an increase in textbook usage and the large majority of 64% who plan no change. Mathematics, probably to a greater extent than other subjects in the primary school, tends to be characterised by a heavy reliance on published textbook schemes. The results of this survey suggest that there is likely to be only a small improvement in this situation.

This phenomenon is not unique to Scotland. An OFSTED (1993) report expressed concern that there was an over-reliance on published schemes in over one third of the classes observed. Johnson and Millett (1996, p55) noted that in most schools,

The planning task was seen as one of reviewing the existing published scheme to see how well it matched with the NC (National Curriculum), rather than using the Order to plan new schemes of work.

### **5.2.8 Teachers' own attainments in mathematics and feelings about the subject**

This section of the questionnaire was included to look at teachers' qualifications in mathematics and to see whether the level of their qualifications was related to their feelings about mathematics and their confidence in their own ability to teach it.

#### **Detailed findings**

For slightly more than half of the sample, the highest level achieved was O-grade mathematics or equivalent. Fewer than half had a qualification at Higher level or above.

When asked to list any post-qualifying award-bearing courses taken in mathematics education, 19 teachers replied, but of those only 6 had taken courses which were specifically in mathematics education. The others had followed, for example, certificate courses in Information Technology, or Infant Education, which had elements of mathematics in them.

Teachers were then asked to describe their own mathematical ability, in relation to their ability to teach mathematics at any level in the primary school. The resulting percentages are in Table 5.22.

*"In relation to your ability to teach maths at any level in the primary school, how would you describe your own mathematical ability? (Please tick one box)"*.

Table 5.22  
*Teachers' assessments of  
their own abilities to teach mathematics  
at any level in primary schools. (n=328)*

excellent	11.0
very good	47.0
adequate	37.5
inadequate	2.1
very poor	0.3
no response	2.1

Table 5.23 shows teachers' preferences for the five main curricular areas in the primary school.

*"Please show an order of preference for the five curricular areas given. Write a 1 against the area you most prefer to teach followed by a 2 for your second favourite, and so on."*

Table 5.23  
*Teachers' subject teaching preferences. (n=328)*

	1	2	3	4	5	no response	overall popularity rating
English Language	36.7	26.9	20.2	9.5	1.8	4.9	1
Environmental Studies	22.6	18.9	29.3	20.7	4.0	4.5	3
Expressive Arts	9.1	11.9	21.0	38.2	15.2	4.6	4
Mathematics	27.4	35.7	18.9	11.0	2.7	4.3	2
Religious and Moral Education	3.0	2.8	7.6	13.1	68.9	4.6	5

This shows that the teachers in this sample preferred teaching English language marginally over Mathematics. Environmental Studies was clearly third in popularity

followed by Expressive Arts and, at a distance, by far the least popular subject with the teachers in this sample, Religious and Moral Education.

A cross tabulation of teachers' perceived ability to teach mathematics at any primary stage (Q31) with their subject teaching preferences (Q32D), shows a strong relationship between the two which is statistically significant at the 0.01 level (Appendix 5.1). A cross tabulation of Question 31 with Question 29, teachers' own mathematics qualifications, showed that only 47% of teachers with the minimum entry qualifications rated themselves as more than adequate in terms of their ability to teach mathematics at any level in the primary school, whilst 78% with Higher Mathematics or above described their teaching ability as more than adequate (Appendix 5.1).

A comparison of teachers' ratings of mathematics as a preferred teaching subject, with teachers' own mathematics qualifications (Appendix 5.1), showed that 60% of those who had the minimum entry qualifications for teaching (O-Grade Mathematics, Lower Mathematics, Standard Grade or Scotvec modules), rated mathematics as either their first or second most favoured teaching subject, whilst 75% of those with Higher Mathematics and above rated it in the same top two categories. These results given in Appendix 5.1 suggest that teachers' success in having achieved beyond the minimum entry requirements in mathematics was being reflected both in their confidence in their ability to teach mathematics and in their subject teaching preferences.

### Commentary

On the one hand it is gratifying that so few teachers considered themselves to be less than adequate in their own mathematical ability, but on the other hand it could be a cause for concern that almost 40% of teachers in the sample described their mathematical ability as only 'adequate' in terms of their ability to teach mathematics at any level in the primary school. This apparent modesty or lack of self-confidence in mathematics was not reflected in teachers' views about mathematics as a curricular teaching area, as shown in Table 5.23.

5.2.9 Attitudes in schools to the *Guidelines* and background information

Summary

The purpose of this section was to discover how teachers' colleagues in schools felt about the *Guidelines*. All questions before this had surveyed the views of the respondents themselves. Those here attempted to use the participants to report on the reactions to the *Guidelines* of a much wider sample of teachers. Other information sought concerned the primary stages taught and the length of teaching experience.

The results of the questions in this section reflected the impression gained in Section 5.2.1, that teachers' reactions to the *Guidelines* were generally positive. This suggests that the sample of teachers used in this survey was not biased since their generally positive views seem to be shared by their colleagues. The possibility exists that there may have been bias at the school level but this can not be tested with the data of this survey. There was also a feeling that schools' management teams were responsible for creating positive attitudes to the *Guidelines* among teachers.

Detailed findings

Question 36 provided data about the stages taught by the teachers involved in the survey. Because of the existence of schools with small numbers of teachers and resulting composite classes, the total of the numbers in Table 5.1 exceeds the sample size, since many teachers taught more than one stage. These details have been discussed earlier in this chapter, but Tables 5.1 and 5.2 are repeated here for convenience.

Table 5.1  
*Primary stages taught by teachers in the survey. (n=311)*

Stages	P1	P2	P3	P4	P5	P6	P7
Percentage of participating teachers at each stage.	13.8	13.4	10.5	13.8	13.2	15.2	20.1

Table 5.2  
*Primary stages taught. (n=311)*

Stages	P1	P2	P3	P4	P5	P6	P7	more than one stage
Numbers of teachers	35	27	19	26	28	24	53	99
Percentages of teachers	10.7	8.2	5.8	7.9	8.5	7.3	16.2	31.8

Given that the researcher had no direct control over which participants were selected to take part in the survey, these numbers represent an acceptable spread across all stages of the primary school.

Most of the teachers involved were very experienced, as shown by the percentages in Tables 5.24 and 5.25

Table 5.24  
*Length of time in teaching. (n=328)*

1 year or less	0.6
2-3 years	4.0
4-10 years	16.5
11 years or more	76.5
no response	2.4

An analysis of the data from question 37, which is shown in Table 5.24, together with the responses to question 2 (Appendix 5.1), suggested that teachers' views about the *Guidelines* did not appear to have any relationship with the length of time they had been teaching. In fact there was no statistically significant relationship between the responses to question 37 and those of any other question. This may have been because the population of the sample in this survey consisted mostly of teachers with more than eleven years of teaching experience. The survey by Williams *et al.* (1998) into Scottish teachers' use of ICT, also found that the ages of teachers were not related to other data in that survey.

Are you hoping to retire from teaching within the next 5 years?

Table 5.25  
*Teachers within 5 years of retiral. (n=328)*

Yes	16.4
No	82.3
no response	1.3

The results of this question were similar to those relating to the teachers' ages in the sense that they did not show any correlation with the responses to any other question. This would suggest that teachers' attitudes to the *Guidelines* were unaffected by the stage of their career and that it is not possible, on the evidence of this survey, to suggest that younger teachers nearer the beginnings of their careers are any more or less enthusiastic about, or receptive to, curricular change than their older more experienced colleagues.

5.2.10 Additional comments

Teachers were invited, in questions 35 and 39 of the questionnaire, to add any other comments about their experiences of implementing the *Guidelines*.

Most of the comments repeated points already made elsewhere in the survey. Of the 199 responses received, the main ones (with frequencies for this section of the questionnaire in brackets) which had been mentioned before were:

- More time needed. (51)
- More resources needed. (39)
- Need for 'schemes' to match *Guidelines*. (17)
- Help wanted with P.S.E. (12)
- More in-service needed. (14)
- No more change wanted. (7)

Additional thoughts which had not appeared previously included:

- the need to make secondary schools aware of 5-14 (6)
- the need for improved primary/secondary liaison (3)
- *Guidelines* contributed to a worthwhile course (8)
- the more structured approach was welcome (3)
- mathematics was the easiest of the guidelines to implement. (3)

### 5.2.11 Conclusions

The *Guidelines* appear to have been well received by most teachers and this positive view was held regardless of teachers' length of experience or stage taught. The reservations expressed tended to be constructive, with requests for more time and resources to help with the proper implementation of what was seen as a welcome curricular change.

The new framework of outcomes, strands, targets and levels was seen as helpful for purposes of planning, assessing, recording and reporting. Amongst the other innovations introduced by the *Guidelines*, the outcome on problem solving and enquiry seemed to have presented teachers with the greatest challenge. The absence of strands, levels and targets in this outcome caused teachers some anxiety, as did the lack of guidance about progression within the outcome.

The request for teachers to reconsider the balance of time devoted to each of the three modes of calculating, namely paper and pencil, calculator and mental, did not appear to be having the desired effect of increasing the attention paid to mental methods. Teachers rather seemed to be increasing their usage of calculators.

The message about greater use of a variety of contexts in mathematics was generally welcomed, although some teachers were not confident of their ability to implement what they saw as a good idea.

Teachers were generally appreciative of the support they had been given to help in the implementation of the *Guidelines*. The most highly valued type of support seemed to have been provided by opportunities to discuss the new ideas in the *Guidelines* within their own schools. More formal support mechanisms, in the form of materials and courses offered by regional authorities and colleges of education as well as documentation from the SCCC, were also appreciated.

The results of this survey show some marked differences from similar surveys carried out in relation to the implementation of the National Curriculum Mathematics in England and Wales. The two most noticeable of these differences concerned firstly, the absence of comment by Scottish teachers about class size as an inhibiting factor in the implementation of the *Guidelines*, and secondly, the fact that teachers in Scotland did not seem, on the whole, to be worried about changes in content to be taught. By contrast, and as noted earlier, Scottish teachers did share with their English and Welsh colleagues a degree of uncertainty about the implementation of problem solving and enquiry into the primary mathematics curriculum.

The overall picture provided by the results of this survey is of a teaching body committed to trying to implement the national *Guidelines* in a professional way. Whilst, in teachers' perceptions, there were some imperfections and inadequacies in this new curricular initiative, and while more time and resources would have been appreciated, most teachers demonstrated a willingness to engage with its new approaches and the ideas contained within it.



## Chapter 6 Results of In-Depth Study

### 6.1 Introduction

This chapter will consider the results of the year-long study carried out with fourteen primary 7 classes in the academic year 1996-97. Most of the data were obtained from work with pupils in all fourteen classes apart from those relating to the pre- and post-tests in which only thirteen schools were involved. This happened because the fourteenth school joined the study at a later date and was inadvertently given a pre-test which was different from that given to all other schools. Consequently, these data were rejected.

There were two hundred and nineteen children in the thirteen classes who took the pre- and post-tests. Since both tests consisted of six items and it was felt that three items at one time were as much as could be expected, four days were used for each pupil and not surprisingly a number of pupils missed one or more of the four days used. For the purposes of statistical comparisons it was decided to omit the records of all those pupils who had missed one or more parts of the two tests. This left one hundred and eighty nine pupils who wrote both parts of both the pre-test and the post-test. Two pupils from each of the fourteen classes were selected as subjects for interviews and observations throughout the year.

The fourteen teachers involved had all volunteered to take part in the study and had been briefed at the beginning of the year by the researcher. Teachers in groups A and B had been given slightly different briefings, as was explained in detail in Section 4.2.7. Teachers of group B classes had been asked to stress the reporting aspect of problem solving and those of group A classes had not. In this chapter the results are discussed and analysed in terms of the whole cohort of teachers and pupils from all schools as well in the two separate groups of teachers and pupils in the group A and B schools.

In the course of interviewing all the teachers at the end of the year, it became clear that two of the teachers in group A had independently chosen to follow a course of action which in some respects was similar to that which had been requested of the group B teachers. A possible effect of this might have been to invalidate, to a certain extent, their designations as members of group A. There was, however, no way of ascertaining the extent to which they had stressed the reporting aspect of problem solving, as group B teachers had been explicitly requested to do throughout the year. The only evidence available to the researcher that they might have done this to any extent was in remarks made by the two teachers in the interviews. This is explained later in this chapter in Section 6.6. In fact, one of these two schools was a small rural school which had only two participating pupils and the other teacher had a composite P5, P6 and P7 class in which most of the children from all three stages were involved. Since most of the pupils in this class were younger than average, their scores on both pre- and post-tests tended to be lower than average.

To examine the possible effects of these two teachers' actions, most of the comparative data in this chapter were considered both without and with adjustments made to take account of the fact that the two teachers had in effect re-designated themselves. These adjustments involved re-allocating the scores of pupils in the two teachers' classes to group B and deducting them from those in group A. In most cases, however, the adjusted calculations did not differ from the originals in any statistically significant way and for this reason the data discussed in this chapter are derived from the results obtained from the original designation of classes.

## 6.2 Results of pre- and post-tests

### 6.2.1. Results for all pupils

These results compare the performances of the one hundred and eighty nine pupils from thirteen schools who took both parts of both the pre- and post-tests. The pre-test was administered in October 1996 and consisted of six items with a maximum possible score of 32. A copy of the test is in Appendix 6.1. The post-test, which is shown in Appendix 6.2, also had six items with a maximum score of 31 and was given in May 1997.

In this section a standard error of measurement has not been calculated for any of the tables although it is recognised that the measurements in many of the tables do have error attached to them. In cases where tests of significant differences have been carried out, these tests do take account of the measurement error.

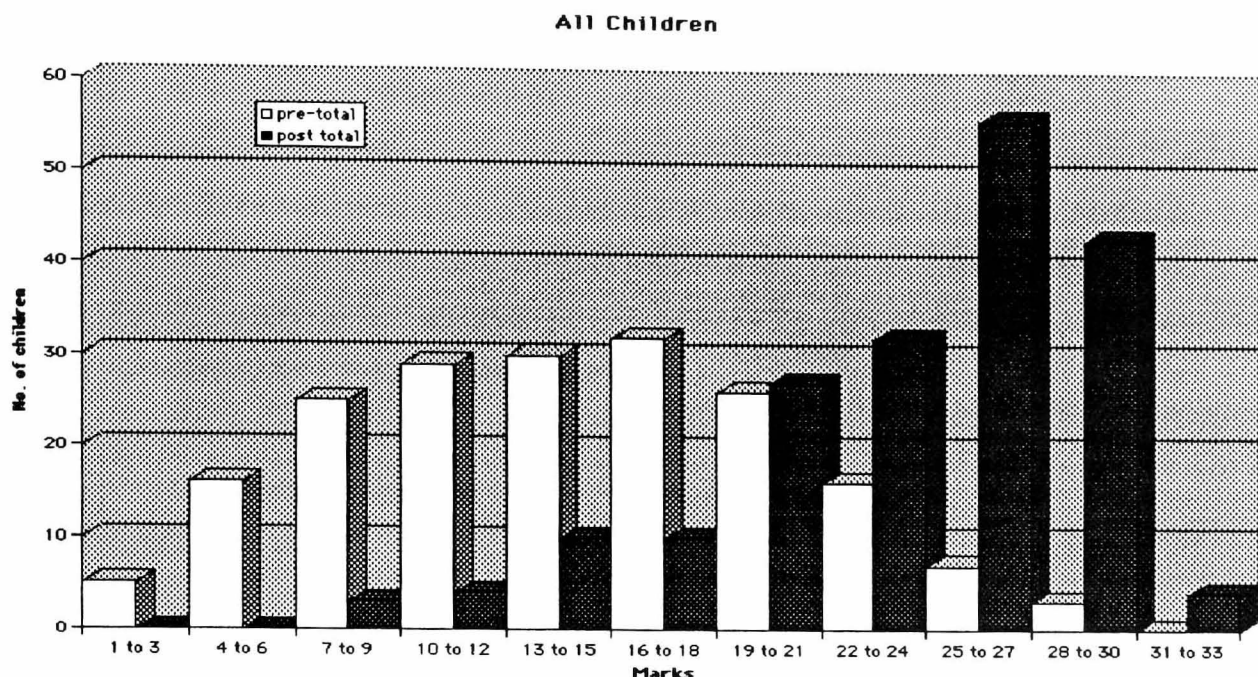
Table 6.1 shows some statistics of the results of the two tests.

Table 6.1  
*Mean total scores on pre- and post-tests*

	maximum	mean	standard deviation
pre-test	32.00	14.37	6.18
post-test	31.00	23.81	5.04

From these statistics it is clear that pupils' performances on the post-test gave rise to higher scores than they had on the pre-test, with a difference of more than nine between the means of the scores on the two tests. This was entirely expected since the pupils had had the benefit of six months' experience of problem solving between the pre- and post-tests and most of the pupils involved had done little, if any, organised programme of problem solving activities before this year. The improvement from pre- to post-test can be seen in Figure 6.1.

Figure 6.1  
*Scores of all pupils on the pre- and post-tests.*

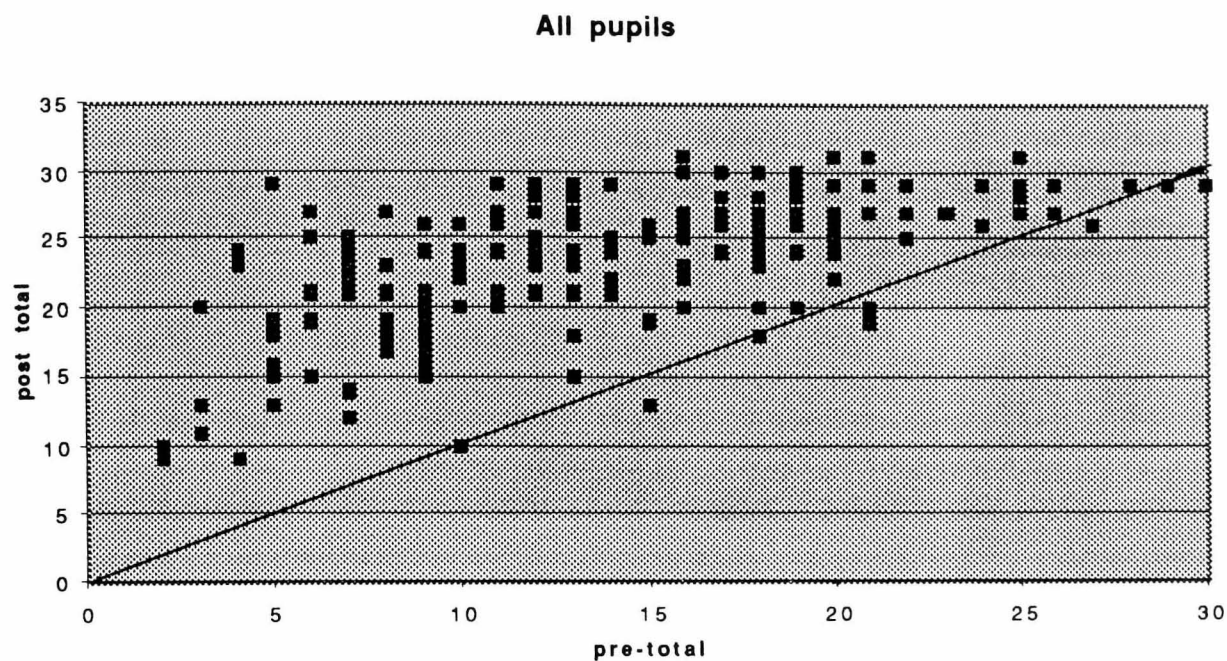


A frequency table of the differences, shows that more than 97% of the pupils improved their scores on the post-test, with only five pupils' scores falling at the end of the year. The pairs of pre- and post-test scores of each of these pupils were 30 - 29, 27 - 26, 21 - 20, 21 - 19, and 15 -13. The first two of these pairs represent excellent scores on both tests and can be accepted as such. The other three show small drops in performance which are more difficult to explain. Since they represent only 2.6% of the whole cohort of pupils, their relatively poor performance on the post-test may be attributed to any of a variety of factors which are impossible to confirm. These might include the individuals' mental, psychological or physical state on the days when they did the post-test. The mean difference for all pupils between the pre- and the post tests was 9.44.

There was a high correlation (0.61) between the scores on the pre-test and those on the post test. This correlation was significant at the 0.01 level.

The distribution of the scores of all pupils is shown in the scattergraph in Figure 6.2.

Figure 6.2  
*The distribution of all pupils' scores on the pre- and post-tests*



This graph provides another view of the improvements of the scores from pre- to post-test. The pupils whose scores dropped between the first and second tests can be seen here, as points below the diagonal, as can a number of students whose scores were low (<10) on the pre-test but high (>20) on the post-test. These latter pupils are identified by the points at the top left of the array and are discussed in some detail in Section 6.2.2.

Table 6.2 shows the correlations between the scores of all pupils on all pairs of items and the pre- and post-test totals on the two tests. Cells which are lightly shaded indicate pairs of items for which the correlation of the scores was significant at the 0.05 level. Those with darker shading had a significance level of 0.01.

Table 6.2  
*Correlations of the scores on pre-test and post-test item pairs by all pupils. n=189*

	pre 1	pre 2	pre 3	pre 4	pre 5	pre 6	pre-test total
post 1	0.25	0.12	0.18	0.19	0.11	-0.05	0.23
post 2	0.33	0.28	0.16	0.23	0.18	-0.03	0.32
post 3	0.17	0.23	0.23	0.21	0.21	-0.05	0.30
post 4	0.33	0.31	0.23	0.30	0.35	-0.01	0.44
post 5	0.16	0.38	0.30	0.44	0.23	0.10	0.46
post 6	0.26	0.34	0.32	0.38	0.39	-0.01	0.50
post-test total	0.40	0.48	0.40	0.50	0.44	0.00	0.65

Table 6.3 shows the correlations between all pairs of items and between all items and the totals on the pre-test and Table 6.4 shows the corresponding correlations on the post-test.

Table 6.3  
*Correlations of the scores on all item pairs on the pre-test by all pupils. n=189*

	pre 1	pre 2	pre 3	pre 4	pre 5	pre 6	pre-test total
pre 1	1.00	0.30	0.26	0.20	0.31	0.05	0.57
pre 2	0.30	1.00	0.27	0.26	0.34	0.10	0.65
pre 3	0.26	0.27	1.00	0.41	0.30	0.04	0.70
pre 4	0.20	0.26	0.41	1.00	0.23	0.01	0.61
pre 5	0.31	0.34	0.30	0.23	1.00	0.13	0.69
pre 6	0.05	0.10	0.04	0.01	0.13	1.00	0.28
pre-test total	0.57	0.65	0.70	0.61	0.69	0.27	1.00



Table 6.4  
*Correlations of the scores on all item pairs on the post-test by all pupils. n=189*

	post 1	post 2	post 3	post 4	post 5	post 6	post-test total
post 1	1.00	0.21	0.04	0.18	0.05	0.24	0.33
post 2	0.21	1.00	0.36	0.39	0.19	0.19	0.59
post 3	0.04	0.36	1.00	0.28	0.29	0.12	0.53
post 4	0.18	0.34	0.28	1.00	0.30	0.33	0.76
post 5	0.05	0.19	0.29	0.30	1.00	0.30	0.63
post 6	0.24	0.19	0.12	0.33	0.30	1.00	0.68
post-test total	0.33	0.59	0.53	0.76	0.63	0.68	1.00

Although a large number of the correlations between pairs of items in Tables 6.2, 6.3 and 6.4 are significant at the 99% level, these correlations are not restricted to pairs of items which share a common strategy. This, together with the fact that the correlations are relatively low, means that we can not infer much from them and also suggests that the strategy used may not be an important factor in determining item difficulty. It may, therefore, be more informative to look closely at items which do not correlate.

The one item in Tables 6.2 and 6.3 which has no statistically significant correlation with any other is item 6 on the pre-test. As can be seen from Table 6.6, which shows the mean facility values of all items, item pre 6 was one of the three most difficult. This fact alone, however, can not account for its low degree of correlation with other items, since the other two most difficult items, pre 3 and pre 4, which were marginally more difficult than pre 6, had generally higher correlations with all the other items, in most cases, as can be seen in Table 6.2, at the 0.01 significance level. The factor which may explain the lack of correlation with other items is that it was a problem which would most readily be solved by use of a Venn diagram to help with the logical deduction necessary for its successful solution. Most, if not all P7 pupils have not met or used Venn diagrams, a fact which could account for its low facility value. It may be that the kind of reasoning required for the solution of this item is completely different from the reasoning needed for all of the other items. The low and often negative correlations shown in Tables 6.2 and 6.3 suggest

that the pupils who were successful at this level and type of logical thinking were less so in the other items. Another possibility was that a certain amount of guessing took place and, although full marks for the item could not be gained without some explanation, it was possible for pupils who guessed to earn some marks. The text of item pre 6 was,

Every red car at a motor show was a French car.  
Half of all the blue cars were French.  
Half of all the French cars were red.  
There were 40 blue cars and 30 red cars.  
How many French cars were neither blue nor red?  
Explain your answer.

Another item which stands out as not displaying the same level of correlation as the others is post 1. This may be due to the fact that it was not only the easiest of all items for all pupils but also that 93% of them scored full marks on it. The fact that so many pupils reached their ceiling on this item suggests that it would not show a high degree of correlation with other items.

### **6.2.2 Comparisons of groups A and B.**

Pupils in group B classes produced better results on six items over both tests whilst group A pupils scored more highly on five of the twelve items. However, these differences are mostly minor and none of them is statistically significant. Table 6.5 gives the maximum possible score and the mean score on each item in raw scores for each group and for all pupils.



Table 6.5  
*Mean scores for test items*

Item	maximum	group A mean n=98	group B mean n=91	all pupils mean. n=189
pre 1	4.00	3.09	3.07	3.08
pre 2	6.00	2.52	2.75	2.63
pre 3	7.00	2.34	2.53	2.43
pre 4	5.00	1.64	1.67	1.67
pre 5	6.00	3.03	3.23	3.13
pre 6	4.00	1.52	1.38	1.46
post 1	4.00	3.89	3.81	3.85
post 2	5.00	4.56	4.56	4.56
post 3	5.00	4.41	4.65	4.52
post 4	5.00	3.96	3.66	3.81
post 5	5.00	3.67	3.88	3.77
post 6	7.00	3.32	3.26	3.29

Table 6.6 shows the mean facility values for each of the twelve test items achieved by both groups and by all pupils, expressed in percentage terms for purposes of comparison. As can be seen from Table 6.7 the pupils in group B classes outscored those in group A classes in both the pre- and the post-tests but in neither case was the difference statistically significant.

Table 6.6  
*Mean facility values for test items in percentages*

Item	group A mean	group B mean	all pupils mean
pre 1	77.3	76.8	77.0
pre 2	42.0	45.8	43.8
pre 3	33.4	36.1	34.7
pre 4	32.8	33.4	33.2
pre 5	50.5	53.8	52.2
pre 6	38.0	34.5	36.5
post 1	97.3	95.3	96.3
post 2	91.2	91.2	91.2
post 3	88.2	93.0	90.4
post 4	79.2	73.2	76.2
post 5	73.4	77.6	75.4
post 6	47.3	46.6	47.0

The mean total scores on both pre- and post-tests with pupils separated into the two groups are shown in Table 6.7 in which raw scores are used. Corresponding frequencies for the difference scores between pre- and post-tests are shown in Table 6.8 for each of the two groups.

Table 6.7  
*Mean scores on the pre- and post-tests*

	maximum possible	group A	group B	all
pre-test total	29.00	14.14	14.62	14.37
post-test total	31.00	23.81	23.82	23.81
differences		9.67	9.20	9.44

These data show only minor differences in the statistics produced from the results of each group's scores and none of these differences is statistically significant. Pupils in group B outperformed their group A counterparts by a small margin in the mean scores of both pre- and post-tests, 14.62 - 14.14 and 23.82 - 23.81 respectively, whilst the improvement shown from pre- to post-test by group A pupils was slightly greater than that achieved by those in group B, 9.67 - 9.20.

Table 6.8  
*Frequencies of differences between post-test and pre-test scores of pupils in groups A and B*

Differences Post - pre-test	Group A			Group B		
	Frequency	Percent	Cumulative percent	Frequency	Percent	Cumulative percent
-2-0	5	5.0	5.1	3	3.3	3.3
1-3	4	4.0	9.2	8	8.8	12.1
4-6	10	10.0	19.4	15	16.5	28.6
7-9	26	26.5	45.9	24	26.4	54.9
10-12	26	28.5	74.5	17	18.7	73.6
13-15	16	16.3	90.8	12	13.2	86.8
16-18	6	6.1	96.9	9	9.9	96.7
19-21	3	3.0	100.0	2	2.2	98.9
22-24	0	0.0	100.0	1	1.1	100.0
Total	98	100.0	100.0	91	100.0	100.0

The scores of nine pupils from group A classes and twelve from group B improved by more than 15 marks each, between the pre-test and the post-test. There is no obvious explanation for these large gains. Of the nine group A pupils, seven came from two schools. The school statistics given in Table 6.13 show nothing remarkable about the results in these two schools (with ID numbers 4 and 7), apart perhaps from the fact that school 4 scored the third highest post-test total of all schools, due, in part, to these improvements noted here. Among the twelve group B pupils in which similarly huge improvements took place, eight came from the four schools 8, 9, 10 and 11 in Table 6.13. In these cases also there is no obvious reason for these improvements. The other four pupils in this category all came from school 13 and belonged to a composite class with pupils at the P5, P6 and P7 stages in it, all of whom followed the problem solving programme. Only the P7 pupils, however, took the pre-test and the post-test. This class had done virtually no problem solving before the start of the year's programme, so this may help to explain their improvements and the fact that their school showed the biggest mean difference between pre- and post test scores (Table 6.13).

At the time the pre-test was given to the pupils, many of them had had little or no exposure to problem solving, so it was not surprising that numbers of pupils scored poorly. It was decided to look more closely at those doing so, so Tables 6.9 and 6.10 were produced to show both the pre- and post-test scores of the pupils who scored less than 8 (< 25%) in the pre-test.

Table 6.9  
*Scores on both tests, of pupils scoring < 8 in the pre-test. Group A schools.*

Pupil	Pre-test score	Post-test score
A	5	18*
B	5	15*
C	3	13*
D	5	13
E	6	21*
F	3	11
G	2	10
H	4	24*
I	4	25*
J	7	25*
K	7	21*
L	7	14
M	4	9
N	4	23*
O	6	25*

Table 6.10  
*Scores on both tests, of pupils scoring < 8 in the pre-test. Group B schools.*

Pupil	Pre-test score	Post-test score
P	7	22*
Q	7	23*
R	5	19*
S	6	19*
T	7	24*
U	7	14
V	5	18*
W	2	9
X	7	12
Y	4	9
Z	7	14
AA	5	16*
BB	6	15
CC	5	29*
DD	6	27*
EE	3	20*

\* denotes pupils who made an above average improvement from pre- to post-test.

In Table 6.9 pupils B-G came from a small rural school and belonged to a composite P5, P6 and P7 class, all of whom the teacher wanted to take part in the study. There is a likelihood that at least some, if not most, of these scores came from P5 or P6 pupils. This may account for their lower starting scores and the fact that some of the improvements shown were below average. Most of the other scores in this table showed gains well above average, as indicated by the single asterisks beside them, with the exception of pupils L and M. There are no obvious reasons why these two scores should be as they

are. It must be assumed that these are from two low attaining pupils who did not derive the expected benefit from the year's exposure to problem solving.

Pupils CC - EE, whose scores are shown in the bottom three rows of Table 6.10 also belonged to a composite P5, P6 and P7 class and are three of the four pupils discussed in relation to the large gains noted in Table 6.8. Although the problem solving programme had also been done with all the pupils in the class in this school, as noted previously, only those pupils who were at the P7 stage had done the pre- and post-tests. This may account for the improvements noted.

The first five sets of scores in Table 6.10 (P-T) represent good gains in performance whilst the rest of the scores in this table, apart from the bottom three just mentioned, all come from pupils in the same class of 30 pupils. Apart from the fact that these pupils were all at the P7 stage, no further information is available about this class, so no inferences can be drawn as to the reasons for these relatively poor results.

The correlations between the scores on the two tests calculated separately for group A and group B pupils are 0.607 and 0.616 respectively. Once again these correlations are high and are significant at the 0.01 level.

The distribution of the pre- and post-test scores of the pupils in each of the groups is shown in the scattergraphs in Figures 6.3 and 6.4.

Figure 6.3  
*Distribution of group A scores on the pre- and post-tests*

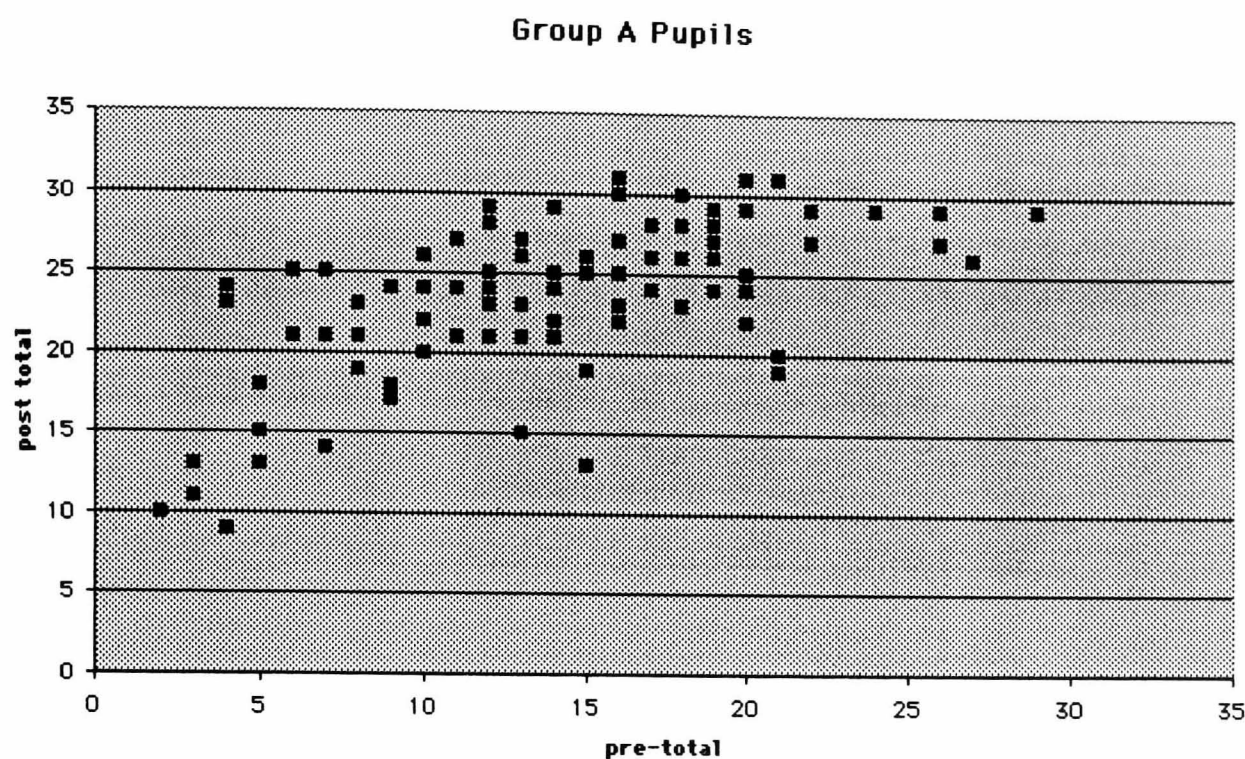
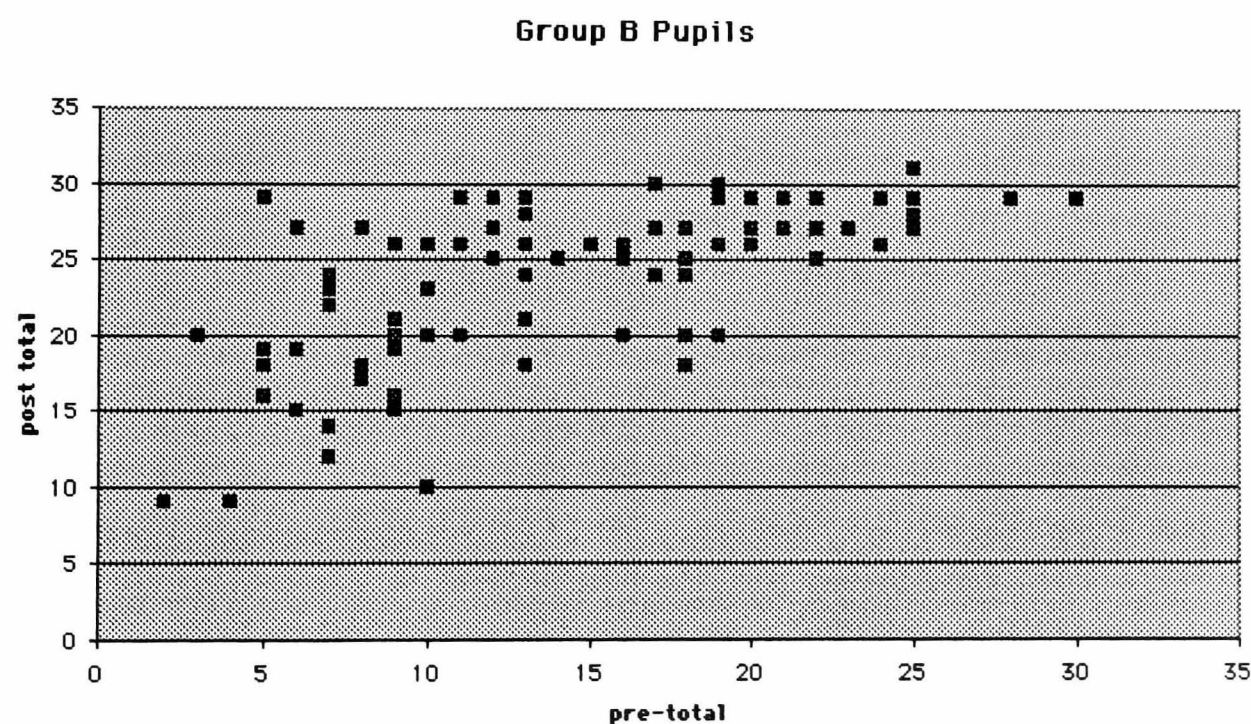


Figure 6.4  
*Distribution of group B scores on the pre- and post-tests.*



These graphs illustrate the general improvement over the two tests. The performances of individual children who performed poorly ( $<10$ ) on the pre-test and well ( $>20$ ) on the post-test can also be noted in the top left hand area of the cluster on each graph. Some of these pupils are the same ones whose scores were given in Tables 6.9 and 6.10 and whose cases were discussed at that time.

A one-way analysis of variance was calculated with the post-test total scores as the dependent variable and the group A/group B categorisation as the main effect (or group). There was no statistically significant difference between the results of the two groups of schools, so the categorisation into groups A and B had no effect on the results.

Tables 6.11 and 6.12 show the correlations between the scores of group A and group B pupils respectively on each pair of items.

Table 6.11

*Correlations of the scores on all item pairs by pupils in group A classes. n=98*

	pre 1	pre 2	pre 3	pre 4	pre 5	pre 6
post 1	0.33	0.17	0.19	0.16	0.06	-0.04
post 2	0.31	0.31	0.28	0.28	0.15	-0.04
post 3	0.19	0.32	0.25	0.15	0.25	-0.08
post 4	0.23	0.31	0.18	0.24	0.23	-0.07
post 5	0.07	0.33	0.26	0.36	0.24	0.15
post 6	0.28	0.32	0.31	0.39	0.31	0.03

Table 6.12

*Correlations of the scores on all item pairs by pupils in group B classes. n=91*

	pre 1	pre 2	pre 3	pre 4	pre 5	pre 6
post 1	0.17	0.09	0.18	0.22	0.15	-0.06
post 2	0.35	0.24	0.05	0.18	0.20	-0.03
post 3	0.15	0.10	0.21	0.30	0.17	0.02
post 4	0.42	0.32	0.28	0.36	0.46	0.05
post 5	0.26	0.42	0.33	0.53	0.21	0.06
post 6	0.23	0.37	0.35	0.37	0.48	0.07

These two tables confirm the anomalous results produced by item pre 6 which were noted and discussed in the previous section in relation to Tables 6.2 and 6.3. Tables 6.11 and 6.12 also highlight, even more noticeably than did Table 6.2, the lack of significant correlation of item post 1 with most other items. This was the item in which 93% of pupils scored full marks. As a result of this, it had a standard deviation of 0.49, which was the lowest recorded for any item. Two other items showed a similar effect in Table

6.12. These were post 2 and post 3 in which 82% and 80% of pupils respectively reached their ceiling by scoring full marks. These items had standard deviations of 1.05 and 1.04 respectively which were, with the exception of item post 1, the lowest recorded. It is clear that these three items, in which such a high proportion of pupils scored maximum marks, would not, for that reason, discriminate between or rank pupils effectively and hence lower correlations with other items could be expected.

### 6.2.3 Analysis of school results

The results from each school on both tests are displayed in Table 6.13. The first column gives the school's identity number and shows whether the school belonged to group A or B. In column 2, N shows the number of pupils in the school who sat both parts of both the pre- and post-tests. The mean scores of the totals scored on the pre-test and on the post-test and the mean of the differences between these two are given in column 3. The standard deviations of each of these three are given in column 4. The correlations between pupils' total scores on the pre- and post-tests are given in column 5. Single asterisks indicate significance at the 0.05 level and two asterisks indicate significance at the 0.01 level.



Table 6.13  
*School statistics*

school ID Group		N	Mean	Standard deviation	Correlation between pre- and post-test totals
1 A	pre-test total	7	13.29	5.59	0.82*
	post-test total		24.00	4.83	
	differences		10.71	3.15	
2 A	pre-test total	2	16.00	0.00	
	post-test total		23.50	2.12	
	differences		7.50	2.12	
3 A	pre-test total	16	9.63	5.31	0.69**
	post-test total		19.00	5.85	
	differences		9.38	4.41	
4 A	pre-test total	25	15.88	5.25	0.57**
	post-test total		26.24	2.55	
	differences		10.36	4.32	
5 A	pre-test total	17	12.71	3.89	0.30
	post-test total		22.82	4.54	
	differences		10.12	5.01	
6 A	pre-test total	14	16.36	5.00	0.64*
	post-test total		25.58	3.25	
	differences		9.21	3.83	
7 A	pre-test total	17	15.59	7.31	0.52*
	post-test total		24.24	5.33	
	differences		8.65	6.41	
8 B	pre-test total	8	15.63	6.59	0.62
	post-test total		26.25	2.76	
	differences		10.63	5.34	
9 B	pre-test total	13	12.92	5.65	0.80**
	post-test total		24.54	4.22	
	differences		11.62	3.40	
10 B	pre-test total	27	10.59	6.17	0.64**
	post-test total		19.37	5.91	
	differences		8.78	5.12	
11 B	pre-test total	20	18.00	4.54	0.63**
	post-test total		25.20	4.23	
	differences		7.20	3.81	
12 B	pre-test total	14	19.57	5.21	0.73**
	post-test total		26.93	2.23	
	differences		7.36	3.89	
13 B	pre-test total	9	13.11	7.25	0.19
	post-test total		26.11	2.93	
	differences		13.00	7.28	

No correlation is given for school 2 since both pupils had the same score on the pre-test, making one of the variables constant. School 3 had the lowest mean total score on the

post-test and the second highest standard deviation of all schools. Both of these results are attributable to the fact that the pupils in this school who took both tests came from the three stages P5, P6 and P7. These are the same pupils whose scores were referred to earlier in connection with the results shown in Table 6.9. The predominance of younger pupils in this composite class produced the lower set of scores and the greater than average standard deviation. The improvement in the class, however, was almost up to the mean of the difference score which was given in Table 6.1 as 9.44.

The three schools whose pupils produced the highest total scores on the post-test were schools 4, 8 and 12 and it was noticeable that, apart from the two-pupil class in school 2, these classes also had the lowest standard deviations of all schools in the post-test totals. These resulted from the uniformly high marks recorded by most of their pupils on the post-test.

The greatest spread of scores on the post-test, represented by a standard deviation of 5.91, was recorded by school 10. This was a school which was previously commented on in connection with some of the results of a number of low scoring pupils recorded in Table 6.10. The existence of these same pupils is once again manifested in the lowest mean post-test score of all P7 classes, 19.37.

Three schools showed no statistically significant correlation between their scores on the pre- and post-tests. These were schools 5, 8 and 13. The latter two were classes of eight and nine pupils respectively, both of which performed almost universally well on the post-test with seven out of eight and eight out of nine respectively scoring at least twenty four out of thirty on the post-test. School 13 also recorded the highest improvement from pre- to post-test. This was the composite class described earlier, in Section 6.2.2, in relation to the data in Tables 6.8 and 6.10. The results from school 5 are harder to explain, although more than half of the class (ten out of seventeen) produced scores in the relatively narrow range of between twenty one and twenty four out of thirty.

As can be seen in Table 6.14, a one-way analysis of variance with the post-test totals as the dependent variable and the school as the factor (or group) showed a significant difference at the 0.01 level. As noted earlier, a similar analysis of variance, with the post-totals as the dependent variable and the group A/group B categorisation as a factor, produced no statistically significant difference between the two groups of schools. Clearly the school had an effect on the results, but the categorisation of schools to one of the two groups did not.

Table 6.14  
*Analysis of variance. One-way group is school*

		Sum of squares	df	Mean square	F	Sig.
Post-test total	Between groups	1390.46	12	115.87	6.02	0.01
	Within groups	3388.06	176	19.25		
	Total	4778.52	188			

### 6.3 Item Pairs Analysis

In order to try to take account of the relative difficulties of the items in the pre- and post-tests, pairs of questions, one from each of the two tests, were selected for scrutiny to form a basis for comparison of pupils' performances from the beginning of the year (Sept '96) in the pre-test, to the end of the year (May/June '97).

The pairs of questions were chosen as being those most likely to be tackled in a similar manner by most of the pupils in the study. For example, question 1 in the pre-test was compared with question 1 in the post-test since they were very similar in structure and depended for their solutions on the pupils reading the clues and by a process of logical reasoning arriving at their solution. Other pairs of questions were selected using similar criteria.

Each of the question pairs were then given to one of six different P7 classes in October 1997. The pupils in these classes had not previously been involved in this study and had done little problem solving before starting their P7 year. In the 1997-98 session most of these P7 classes were being taught by the teachers involved in the study the previous year, which for them had been their first year of seriously attempting to teach problem solving. In all cases the teachers were working in schools where there had not up until this time been a strong emphasis on problem solving.

Each of the two questions in the pair was assigned the same maximum mark and the pupils' work was graded using similar marking schemes. When the six pairs of questions were administered to the six classes, half of each class was asked to do the second question first in each case whilst the other half attempted them in the normal order. This was an attempt to minimise the appearance of any possible 'fatigue factor' which might have reduced pupils' performances on the second question and alternatively the effects of a possible 'warm-up' factor, which might account for enhanced performance on the second of the two questions.

Pupils' performances on each of the question pairs allowed comparative measures of difficulty to be calculated for each pair of items and for a corresponding expected score on each of the post-test items to be estimated. The actual scores attained on the post-test items in every case exceeded the performance expected from the item pairs analysis, by amounts ranging from 15% to 124%. It seems reasonable to suggest that the year's programme of problem solving experiences which the pupils had undertaken could have accounted for a part of this measured improvement.

When the performances of the two separate groups were analysed in this context, it was found that the group A pupils in most cases had exceeded the expected level of performance by amounts greater than those of their group B counterparts, but only by small amounts which were not statistically significant.

## Commentary

The item pairs comparison and analysis described above was conducted after the main data collection had taken place. Its purpose was to provide a model for measuring the improvements shown by pupils in problem solving over the course of one school year. Comparison of the actual improvements with the expected improvements, which were calculated on the basis of the item pairs comparisons conducted, provided, in this case, a measure which could be used to determine improvements or deteriorations in individual pupils', groups' or whole class' problem solving skills over any given period.

In retrospect, it has been recognised that an item pairs comparison exercise such as this should have been conducted in advance of the main study with pupils not involved in it. The pairs of questions could then have been used to compose the pre- and post tests in a more carefully structured way than was done in this study. Such tests would consist of, say, six items, each of which might depend for their solution on the use of a different strategy. Each of the post-test items would then match each of the pre-test items with regard to the strategy which most pupils would be expected to use and each pair would have been compared for levels of difficulty. Scores made by pupils on such a post-test could be more reliably used for measurements of improved performance in problem solving. This was not done early enough in this study, with the result that not all of the post-test items were able to be used in the items pairs comparison exercise.

It is recognised that there are some sophisticated approaches to determining item difficulty, such as those developed in the 1970s by Rasch (Wright and Stone, 1979), but it was felt neither necessary nor worthwhile to employ such complex processes in this study.

## 6.4 Pupil interview and observation analysis

### Term 1

As noted previously in Chapter 4, two pupils from each class involved in the study were interviewed and observed during the process of solving two problems at two points during the year of the study, in November 1996 and later in March 1997.

The statements of the two problems given in November 1996 were:

**The Slimy Worm**

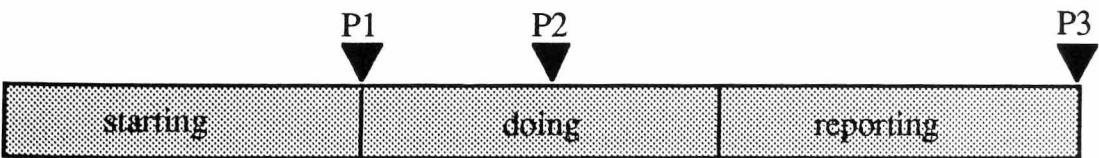
A slimy worm wants to climb to the top of a tower of ten blocks. It can climb four blocks in an hour. But then, because the climbing makes it very tired, it must sleep for an hour. While it is asleep it slides back down three blocks. How long will it take the slimy worm to reach the top of the ten block tower?

**Sally's Marbles**

When Sally puts her marbles in groups of five, she has one marble left over. When she puts her marbles in groups of six, she has one marble left over. She has less than forty marbles. How many marbles does she have?

As explained in Section 4.2.9, these problem solving sessions were observed in terms of each one having three constituent parts or 'episodes', the 'Starting', 'Doing' and 'Reporting' phases of the framework described in the *Guidelines*. At three points during the problem solving experience, previously described as 'executive decision points', the pupils were invited to respond to the researcher's request to describe their thoughts. These executive decision points, P1, P2 and P3, were illustrated in Figure 4.3 which is repeated here for convenience.

Figure 4.3  
*Graph showing pupil observations and intervention points.*



Each pupil response elicited by the researcher's intervention was analysed and coded 1, 2 or 3, according to the extent to which it provided evidence that metacognitive thinking was taking place, especially the kind of metacognition referred to by Flavell (1976) as "regulation or orchestration" of the pupil's cognitive processes. The numerical code given

also reflected the extent to which the oral response matched the problem solving strategy observed by the researcher.

The first intervention by the researcher asked the pupils how they thought they would begin to solve the problem. This was done when it was evident that the pupil had read the problem and was ready to begin some written attempts at a solution. This probe sought to assess the level of metacognition taking place at the 'Starting' phase of the problem solving cycle. Responses which were predominantly descriptive or narrative were coded with a '1'. These included responses which:

- paraphrased the information given in the statement of the problem. e.g.  
It tells you that if she puts them in groups of five, she still has one left over and if she puts them in groups of six she still has one left over.  
Well it said it could climb up 4 blocks in an hour, it sleeps for an hour and it slides back down 3.
- gave numerical details of calculations done mentally, e.g.  
It goes up 4, sleeps and slides down 3, so it takes 2 hours to go one block, so it takes 20 hours to get up.
- described actions to be taken, e.g.  
I'm trying to work out what numbers are in the 6 times table and the 5 times table.

Whilst these types of responses illustrate some ability to describe or monitor the cognitive processes which either had taken or were about to take place, they do not suggest that the pupils are thinking beyond the numbers in strategic terms, are evaluating the likelihood of success or are selecting certain strategies to get started into the problem. In this sense they are not taking what Schoenfeld (1981) described as 'managerial' decisions.

Responses which were coded with a '2' were also descriptive but showed in addition some degree of strategic awareness and included those which:

- referred to both reflective thoughts and arithmetical actions, e.g.  
In my head I'm using my tables for 6 to see if there's one where the 5 and the 6 can go, sort of together.

I'm not sure what the sum might be but I think I'll have to do some er....what's the word....dividing.

- described initial and planned subsequent actions, e.g.

I'm drawing the groups of marbles starting with the fives, then I'm going to see if the same number goes into the sixes.

To begin with I'm just putting down the information.

I'm going to draw the block and show the worm going up it and then down it. Then I'll count the hours.

Responses which suggested that some metacognitive processes were being used were coded with a '3'. Examples of this type were those which:

- referred explicitly to a problem solving strategy, e.g.

I'm going to draw a diagram.

I'm going to draw a picture to start off with.

- showed that pupils were thinking in strategic ways, e.g.

I'm guessing that it's a number in the five and six times table and I'll see if that does it.

If I draw a picture showing its up and down movements, that should help.

The pupils' responses to the second intervention by the researcher, which took place, as shown in Figure 4.2, during the 'Doing' phase of the solution, were also analysed in a similar way. At this point they were asked to explain what they were doing and where they were at in the solution of the problem.

When the pupils had clearly finished the problem, regardless of whether or not a correct solution had been attained, they were asked to explain what they had done which had helped or allowed them to solve the problem. On this occasion the researcher did not use the word 'strategy' which some of the pupils may not have met, but the question was intended to assess their awareness of, or familiarity with, one or more problem solving strategies. Once again, responses were coded in the same way. This coding was done by the researcher working alone, and whilst the validity and reliability of the coding could have been improved by using one or more additional coders, this was not possible in the context in which the researcher was working. However, in an effort to increase the reliability of the coding it was done as a 'blind' exercise in which the pupils whose



protocols were being analysed and coded were not associated with their schools or with either group A or B. In addition the exercise was repeated in a similar way after an interval of several weeks. This happened in both terms 1 and 2 and an 88% agreement between the first and second codings was achieved. Whilst this could not be considered comparable to the normally accepted intercoder reliability measure of 85%, it was felt to be satisfactory in the circumstances. No check of the validity of the coding was done and the categorisation of the protocols into the 1, 2 and 3 codes was that of the researcher only.

When transcripts of the pupils' interviews were analysed and the coding of the responses completed, a summary table was completed for each school showing the total of each category of response given by each pupil for both problems. A sample is shown in Table 6.15.

Table 6.15  
*Sample table for recording types of responses given by two pupils to two problems*

School	Group	Starting			Doing			Reporting		
Name	A	1	2	3	1	2	3	1	2	3
Pupil X problem 1			x			x				x
problem 2			x		x					x
Pupil Y problem 1		x			x				x	
problem 2				x		x				x
	Total	1	2	1	2	2	0	0	1	3

This table shows the responses of two pupils X and Y in one school, which was one of the group A schools, in the three phases of each of problems 1 and 2, The Slimy Worm and Sally's Marbles. In all cases each pupil had only one response recorded for each phase of the problem, although occasionally they offered more than one response. In such cases the response which was rated more highly by the researcher was the one recorded. The totals of each category in each of the three phases are given in the bottom row of the table.

This was done for thirteen of the fourteen schools. The interviews in the fourteenth (a group B) school failed to record and consequently are not included in the final totals. The totals of all the responses are given in Table 6.16. Since each pupil had one response coded for each phase on each of two problems, a total of twelve responses were recorded for each pair of pupils. Table 6.16 shows three totals, one for group A classes, one for group B classes and one for both sets of classes combined.

Table 6.16  
*Numbers of pupil responses in categories 1, 2 and 3. Term 1.*

codes	Starting			Doing			Reporting		
	1	2	3	1	2	3	1	2	3
group A schools	17	10	1	17	11	0	15	9	4
group B schools	11	8	5	15	8	1	14	1	9
all schools	28	18	6	32	19	1	29	10	13

From this table it is clear that in most cases category 1 responses were the most common and category 3 types were least common. The exception on this occasion was in the responses given by the children in the group B classes at the reporting phase of the problems when category 3 type responses appeared more frequently than category 2 types. At this point it might be instructive to recall the request made to teachers of the group B classes at the outset of the study and previously described in Section 4.2.7 and in Appendix 4.6. This was to encourage pupils in these classes to describe orally the strategies or processes they had used to solve their problems and subsequently to note this in writing on the reporting form provided for this purpose. Teachers of the group A classes had not been given this request, though it does not necessarily follow that they did not do this. Teachers' behaviour in their own classes was one of the variables in the study over which the researcher had virtually no control beyond the original instructions and requests given.

It could be tempting to attribute the greater proportion of metacognitive responses given by group B pupils at the 'Reporting' phase to the effects of the encouragement to

verbalise their strategies received by the pupils producing these responses. Two factors, however might suggest some caution before making such an assumption. The first is that these interviews and observations took place in term one, before the pupils involved had experienced very much exposure to problem solving and it would have been surprising for the teaching experienced by group B pupils to have produced such noticeable differences at such an early stage. The other factor relates to the possible lack of reliability of the coding procedures used.

The total numbers of each category of response given by the two groups of pupils, with the number of responses in the three categories 1, 2 and 3 aggregated over the three phases, are shown in Table 6.17

Table 6.17  
*Observed frequencies occurring in each category 1, 2 and 3 by both groups of pupils. Term 1.*

	1	2	3	row totals
Group A	49	30	5	84
Group B	40	17	15	72
column totals	89	47	20	156

The data in this table show that 9 more pupils in group A gave category 1 responses and 10 fewer gave category 3 responses compared to those in group B. To consider these data further a chi-squared calculation was done which gave a value of 8.88. With 2 degrees of freedom this is significant at the 5% level, so the null hypothesis that there were no differences between groups A and B can be rejected at this level.

When the two groups' performances on each of problems 1 and 2 were analysed separately, no statistically significant differences were found between the results of the two groups on either of the two problems.

To establish the extent to which the two problems yielded similar responses the data in Table 6.18 were compiled and analysed. These show the responses of all pupils on problems 1 and 2.

Table 6.18  
*The performances of all pupils on problems 1 and 2*

All pupils	1	2	3	row totals
problem 1	51	23	4	78
problem 2	38	24	16	78
column totals	89	47	20	156

In general, pupils' responses to problem 2 showed a slight tendency to be more metacognitive than those given to problem 1 with 13 fewer 1s, 1 more 2 and 12 more 3s being recorded in problem 2. These differences were not expected as the two problems were done simultaneously. While one pupil of each pair did problem 1, the other did problem 2. Then they exchanged tasks, so it is not possible to attribute the improved performance on problem 2 to the experience and practice gained doing problem 1 since half of the sample did problem 2 first.

A chi-squared test applied to the results in Table 6.18, to test the null hypothesis that the proportions of 1s, 2s and 3s scored in each of the two problems should be the same, yielded a result of 9.12, so the null hypothesis that the two problems would produce the same results, could be rejected and the differences are significant at the 95% confidence level.

## Term 2

The same interview and observation procedure was used in March 1997 with the same pairs of pupils. Two different problems were given and the pupils were reminded of what would happen while they were working on them. The statements of the two problems given were:

### **Disco Dancing**

At the P7 disco three boys were dancing with three girls. Each boy wore a different coloured shirt. One was blue, one green and the other was red. The three girls wore the same three colours. When they were all dancing, the boy in red danced next to

the girl in green and he noticed that each one of them was dancing with a partner dressed in a different colour. What colour is the partner of the girl in red wearing?

**The Netball Competition**

Three schools are having a netball competition and each school is entering two teams. Every team in the competition has to play every other team just once. How many netball games will there be? How many games will there be if four schools enter the competition?

As in term 1, the responses were coded using the same coding procedure. Table 6.19 shows the observed frequencies of each category of responses by both groups of pupils. In term 2 all fourteen schools were involved.

Table 6.19.  
*Numbers of pupil responses in categories 1, 2 and 3. Term 2.*

codes	Starting			Doing			Reporting		
	1	2	3	1	2	3	1	2	3
Group A schools	14	12	2	14	12	2	13	12	3
Group B schools	8	16	4	8	17	3	5	10	13
all schools	22	28	6	22	29	5	18	22	16

Once again there was a greater proportion of category 2 responses among those given by the group B pupils than among those in group A. In the starting and doing phases of the problem solving activity category 2 responses predominated in the group B schools and in the reporting phase, category 3 responses were most common. Category 1 responses were more common among group A pupils in all three phases. The results of aggregating the responses over the three phases are shown in Table 6.20.

Table 6.20  
*Observed frequencies occurring in each category 1, 2 and 3 for both groups of pupils. Term 2.*

	1	2	3	row totals
Group A	41	36	7	84
Group B	21	43	20	84
column totals	62	79	27	168

A chi-squared calculation of 13.34 resulting from the number of responses provided by the two groups of pupils in each of the three categories, as shown in this table, allows a rejection of the null hypothesis that the proportions of scores made by both sets of pupils in each category should be the same, at a 99% confidence level.

On this occasion (in term 2) when the scores on the two problems were computed separately no statistically significant differences were found (with chi-squared = 1.12) between the responses to problems 1 and 2.

These results are less surprising than the corresponding result found in the term 1 data in which significant differences between the scores on the two problems were found. There were also no significant differences between the performances of each group on the two problems.

A comparison of the scores of both groups of pupils from term 1 to term 2 was done to see whether the proportions of responses at the higher metacognitive levels had increased over the course of the year and whether this had occurred equally over both groups. The results of this comparison are shown in Tables 6.21 and 6.22.

Table 6.21  
*Observed frequencies of group A pupils scoring 1, 2 and 3 in terms 1 and 2.*

Group A pupils	1	2	3	row totals
Term 1	49	30	5	84
Term 2	41	36	7	84
Column totals	90	66	12	168

As can be seen from this table, in term 2 a bigger proportion of group A pupils recorded category 2 type responses and a smaller proportion category 1 than in term 1. There was also a small proportional gain in those recording category 3 responses. However these differences were not statistically significant with a computed chi-squared value of 1.6. Table 6.22 shows similar data for group B pupils.

Table 6.22  
*Observed frequencies of group B pupils scoring 1, 2 and 3 in terms 1 and 2.*

Group B pupils	1	2	3	row totals
Term 1	40	17	15	72
Term 2	21	43	20	84
Column totals	61	60	35	156

Amongst this group of pupils there was a similar pattern of a reduction in the number of category 1 responses recorded, with a corresponding increase in category 2 responses. The level 3 responses also showed an increase from term 1 to term 2. The differences in Table 6.22 between term 1 and term 2 were statistically significant at the 0.01 level with a chi-squared value of 17.03.

On the basis of these figures, which represent the evidence of observation and interviews, it is possible to conclude that group B pupils were demonstrating a significantly greater degree of metacognitive thought in term 2 than they had done in term 1. It can also be stated from the evidence in Table 6.20, with a 99% confidence level, that group B pupils outperformed their group A counterparts in term 2 as they had done in term 1. Taken at face value, this evidence could suggest that the experience of group B pupils was effective in increasing their metacognitive awareness. All of these conclusions, however, should be treated with some caution because the degree of rigour with which the categorisation was done was limited by the context in which the researcher was working. This issue will be explored further in Section 7.12. The data of this section will be considered together with other qualitative data from the following two sections of this chapter to make some general conclusions.

### 6.5 Pupil year-end interview results

In June 1997 at the end of their P7 year all 28 pupils who had been the subjects of the interviews and observations throughout the year were interviewed by the researcher. The interviews were structured and the pupils' responses were taped and transcribed. Most of

the questions elicited one answer from each pupil, but occasionally more than one response was forthcoming. Such additional responses were accepted without being encouraged or discouraged.

The questions asked and the responses given were as follows.

*1.What have you learned about problem solving this year?*

This question elicited different responses from the two groups of pupils interviewed. Ten of the nineteen responses from pupils in the group B classes referred to having learned a number of strategies, whilst only two from the group A classes mentioned strategies explicitly. Group A pupils offered a much wider range of responses than their Group B counterparts, which suggested that although they did not seem to be familiar with the same range of strategies as group B, they had acquired positive attitudes to problem solving and an awareness that attributes such as effort, patience and perseverance can be helpful. The results are shown in Table 6.23.

Table 6.23  
*Pupils' responses to question 1, showing what they had learned.*

Group B schools	<i>f</i>	Group A schools	<i>f</i>
More and different strategies	10	More and different strategies	2
How to work out different problems	2	The answer is not always what you think or what it seems	1
I got better at it	4	To concentrate & have patience	1
How to use your brain	1	To think about it before writing	2
Thinking of things in other ways	1	To keep reading over the question	2
How to write things down	1	To persevere	3
		To think positively	1
		To try your hardest	1
		The problems get easier	1
		How to tackle difficult things you don't know how to do	1



The evidence in Table 6.23 was gratifying to the researcher on two counts. It suggests that the efforts of teachers of the group B classes in emphasising the importance of strategies seem to have been effective, but it also shows that teachers in group A classes, whose efforts had been channelled in a slightly different direction, had also been effective in instilling appropriate attitudes in their pupils.

*2. Are you more confident about problem solving than you were last September?*

This question was answered in the affirmative without hesitation by all pupils in both sets of classes.

*3. If you are stuck when you first read a problem, what do you do?*

The most noticeable difference in the two sets of responses to this question was the fact that more than one in three of the pupils in the group A classes, as opposed to one solitary pupil in a group B class, were prepared to ask either a neighbour or the teacher for help. The results are shown in Table 6.24.

Table 6.24  
*What pupils did when they were stuck. Responses to question 3.*

Group B schools	<i>f</i>	Group A schools	<i>f</i>
Read it again	7	Read it a few times	5
Try different approaches	2	Try a different strategy	3
Look for facts to help you	1	Think about it	3
Ask the teacher	1	Ask the teacher	5
Write it down on paper	1	Ask a neighbour	4
Keep trying to work it out	1	Draw a diagram	2
Look for a strategy then another	1	Try it in my head	1
Use the information given	1	Make a chart or a list	1
Do something that makes sense	1		

It is encouraging to note that so many pupils, as shown in Table 6.24, showed an awareness of the need to pause, reflect, or go back and re-read the question if they were stuck. The range of ideas to get out of the state of being stuck or what Roberts and Erdos

(1993) refer to as an 'impasse' also demonstrates a degree of metacognitive knowledge that there are a number of possible alternative solution strategies. The belief of the 'impasse'-based theorists (e.g. VanLehn, 1991) is that metacognitive processes are almost always activated by the failure of a particular solution strategy to produce a helpful next step. Although these ideas have not overtaken the more sophisticated theories of strategy development (Roberts, 1991; Schoenfeld, 1987b), it is still a reasonable assertion that the decision to adopt an alternative course of action when stuck can correctly be described as a metacognitive action. In this sense the pupils in this study were demonstrating a use of metacognitive processes.

#### *4. If you try something and it doesn't work, what do you do?*

The responses given by nine pupils in each of the two groups confirm a point made in the next section by their teachers, namely that most pupils were much more aware of a variety of strategies that they could try in the event that the first one did not lead anywhere. The responses given by group B pupils are all very sensible and represent precisely the kinds of actions which their teachers would have been trying to persuade them to follow throughout the year.

Group A pupils also produced an encouraging set of responses though they tended to specify particular strategies which they had probably found useful in their problem solving during the year. All responses are listed in Table 6.25.

Table 6.25  
*What pupils do when one approach to a problem doesn't work. Responses to question 4.*

Group B schools	<i>f</i>	Group A schools	<i>f</i>
Try another way/strategy	9	Try something different	9
Go over it again	4	Ask the teacher	1
Read the question again	1	Read it again	1
Go deeper into your mind	1	Draw a picture	1
		Draw a diagram	1
		Start again	1
		Leave it then come back to it	1

*5. What strategies have you learned about?*

The two sets of responses to this question, not surprisingly were quite different, with pupils from group B classes, whose teachers had been briefed to encourage actions which would build their pupils' recognition of and familiarity with strategies, offering 48 responses. Eighteen strategies were given by group A.

In all of this study the data in the responses to this question provided the single most convincing piece of evidence that the actions taken by the teachers in the group B classes had been effective in increasing pupils' knowledge about strategies and how to use them. The numbers shown in Table 6.26 would seem to confirm one of the research hypotheses, i.e. that by asking pupils to think about and describe their solution processes through an appropriate reporting mechanism, teachers can expect their pupils to develop a greater awareness of a range of problem solving strategies than they might have done without being asked to do this.

Table 6.26  
*Strategies reported to have been used by pupils. Responses to question 5.*

Group B schools	<i>f</i>	Group A schools	<i>f</i>
Draw a diagram	8	Draw a diagram	3
Work backwards	6	Work backwards	1
Make a table	5	Make a table	1
Guess & check	5	Guess and check	1
Use logical thinking	5		
Draw a picture	4		
Make an organised list	3	Make a list	1
Make a chart/graph	3	Use charts or graphs	5
Trial and improvement	2		
Look for a pattern	2	Look for a pattern	2
Write things down	1	Write out the working	2
Act it out	1	Use actual people	1
Use what you know	1	Elimination	1
Experiment with numbers	1		
Use the strategy list on the wall	1		

*6. What is the hardest part of problem solving for you?*

There was a large measure of agreement amongst all pupils in their responses here, with 12 group B pupils and 10 group A pupils identifying the two most difficult aspects of the problem solving process as "deciding how to get started" and "explaining how you did it". Some of the other responses listed separately, such as "choosing a strategy", "making sense of the information", "interpreting the question" and "working out how to do it", may simply have been other ways of expressing similar difficulties. Table 6.27 shows the responses to this question.

Table 6.27  
*What pupils found hardest about problem solving. Responses to question 6*

Group B schools	<i>f</i>	Group A schools	<i>f</i>
Deciding how to get started	9	Deciding how to get started	5
Explaining how you did it	3	Explaining how you did it	5
Choosing a strategy	2	Working backwards	2
Reading it (dyslexic pupil)	1	Interpreting the question	1
Making sense of the information	1	Finding a pattern	1
		Working out how to do it	1

7. *What, if anything, do you like about problem solving?*

Once again there was general agreement between the two groups, with a few negative responses appearing among those from group A pupils. The general feeling was that they enjoyed the satisfaction of having successfully confronted a challenge which demanded some thought. The views expressed here suggest that the teachers involved in the study had delivered a successful problem solving course which had stimulated and challenged their pupils whilst still offering most of them the experience of achievement and success. Table 6.28 shows the responses to this question.

Table 6.28  
*What, if anything, pupils liked about problem solving. Responses to question 7*

Group B schools	<i>f</i>	Group A schools	<i>f</i>
Satisfaction of solving them	5	The feeling of achievement	3
I just like it	3	I just enjoy it	1
It makes you think	3	The challenge	1
I prefer it to maths	1	Solving them	1
You don't have to use the textbook	1	Doing the fun ones	1
Working backwards problems	1	Solving tricky ones	1
Using a diagram	1	I don't particularly like them	1
		I hate it	1
		Murder mysteries and codes	1
		Nothing, but I don't mind doing them	1
		They're not too hard	1

8. *When you finish a problem, does your teacher ask you to explain how you did it? If 'yes', is this always? sometimes? in writing? orally?*

The reason for including this question was to ascertain the extent to which, in the pupils' views, the request made to the teachers in group B had been followed. The responses of the group B pupils suggest that it had, with all fourteen of them saying that they always had to explain how they did it, with eight of them always having to do so in writing and six always orally. Within these two groups some were sometimes also required to respond in the other way. The pupils in group A offered fewer detailed responses with five of them asserting that they did not have to explain their method, as Table 6.29 illustrates. These responses once again suggested that the different teaching given to the group B pupils had been effective in encouraging a description of the method, process or strategy used.

Table 6.29  
*How pupils reported their problem solving methods of solution.  
Responses to question 8.*

	Group B schools <i>f</i>	Group A schools <i>f</i>
Sometimes orally	4	4
Sometimes written	5	5
Always orally	6	0
Always written	8	0
Not asked to report	0	5

In general the results of these interviews were very positive. They provided evidence that most pupils had developed positive attitudes to the subject as well as a growing confidence in their own problem solving ability. They had clearly gone beyond the stage of initial emotional stress which, according to McLeod (1989), was often experienced by children in their initial attempts at problem solving. The growing confidence expressed by all pupils in the study was encouraging to the researcher, since this confidence would, according to the findings of Meyer and Fennema (1988), have a positive effect on their abilities to solve problems.

Pupils also demonstrated an appreciation of the need for personal attributes such as patience and perseverance. A thorough knowledge of the most common problem solving strategies was shown along with an ability to describe them by name, especially but not exclusively amongst group B pupils.

An acceptance of being stuck and knowing a number of tactics to deal with this state is an integral part of learning to become a problem solver and the pupils in these responses had shown that they had this awareness.

From the researcher's point of view, the answers given by the pupils in these interviews provided encouraging and convincing evidence that the teaching experienced by approximately half of the pupils (i.e. those in group B) had had a noticeable effect on their abilities to identify, discuss and describe a wide range of strategies. The findings of these interviews can now be used to help convince teachers of the desirability of persuading their pupils that the attainment of a solution is not the only goal of problem solving. Also important is the skill of being able to describe the process or strategy used to attain that solution, especially if the process or strategy can be transferred successfully to problem solving in other contexts, and if transfer is, or may be, facilitated by skills of recognition and description.

## **6.6 Teacher year-end interview results**

The following is a collation of interviews held with participating teachers at the end of the year of the study. Thirteen out of the fourteen teachers were interviewed, six from group A schools and seven from group B schools. The fourteenth teacher, from a group A school, was off work at the time of the interviews due to a family bereavement.

The responses to each question are summarised in two sections, the first for group A teachers and the second for group B. Noticeable differences and similarities in responses given will be commented on.

*1. Have your class improved as problem solvers this year?*

All thirteen teachers in both groups answered this in the affirmative with no qualifications. Whilst it is possible that the unanimity of these responses may be attributable to the Hawthorne effect, the responses of pupils, as noted in the previous section, tend to confirm these views.

*1(a). If 'Yes' in what ways?*

Group A

Three teachers commented on their pupils' increased confidence in problem solving and an equal number referred to the fact that they knew and were able to use some strategies.

They're more confident. They know some of the strategies now that they can use and they do try them out. It's no longer a case of, "I don't know what to do" and then just giving up. They've got ideas there that they can try to work with.

Other teachers referred to the fact that pupils were much quicker at the end of the year than they had been previously.

In the beginning they constantly came to me saying, "I have been at this for ages (about ten minutes) and I am stuck," and I tried to tell them that it was not one of those things that can be done in ten minutes and that it might take them an hour. If you look at their sheets they might have one that they had been on for days, whereas now they might do two or three in the allotted time during the day.

One teacher noted that the higher attainers in her class had definitely improved, but said that the pupils with reading difficulties still struggled.

Group B

Teachers in this group also noted the increase in pupils' confidence and willingness to have a go and try something, rather than panicking and giving up. Three teachers agreed with the teacher in group A referred to above, noting that pupils at the top and middle attainment levels had definitely improved, but they felt that they had seen no improvement at all in their lowest attaining pupils. In direct contradiction to this though, one teacher noted that,



I would say the biggest improvement is perhaps with those in the bottom group, rather than those in the top group. They now realise that they can make an attempt even though they don't get the proper result. They have a go and keep trying.

Another agreed with this and said that,

There is a slight improvement with the low attainers - they will make an attempt now where they didn't know where to start before.

The teachers in this group also commented on the fact that their pupils seemed to have more ideas of how to start and had acquired a knowledge of some strategies. There were no marked differences between the responses of the two groups apart from the fact that four teachers in group B made reference to the different levels of improvement amongst pupils of different attainment levels, albeit with contrasting views.

*2. At the beginning of the year were your pupils aware of any problem solving strategies? If 'Yes' do you know which ones?*

#### Group A

Five out of the six teachers of the group A classes said that their pupils were not really aware of strategies before the start of this year. Even if they were, one teacher remarked, they certainly were not familiar with the associated terminology. These responses were not surprising as most of the teachers who had volunteered to be part of the study at the outset had done so because they had done little or no teaching of problem solving previously. There was a possibility that their pupils might have done some problem solving in previous years, since most of the teachers were meeting pupils in their P7 year whom they had not taught before. This meant that the teachers had to find out from their pupils what they knew about problem solving in general and about strategies in particular.

I asked them if they knew about strategies such as 'making a list', and they said they did not and thought that maths was multiply, divide, add and take away.

The only teacher who claimed that her A class had met strategies previously said they had done so in the context of the particular mathematics textbook they had been using, in

which they had met what she described as 'word' problems related, for example, to aspects of measurement. In instances such as these they had been encouraged to draw pictures.

### Group B

Most of the teachers of the group B pupils reported a similar lack of previous exposure to the idea of problem solving strategies.

At the beginning of the year they weren't aware of strategies. It was random and I think they tried any maths they had learned. They now tend to look at the chart and are prepared to try one or two of the strategies.

The reference to a 'chart' in this teacher's response relates to one of the suggestions which the group B teachers had been given, that pupils might be helped towards a familiarity with strategies by keeping a wall chart on which a cumulative record of strategies could be kept as they were being met, used and discussed by the pupils.

Of the two teachers who felt that their B classes did have some knowledge of strategies, one felt that they had strategies in mind but lacked the language to describe them. A consequence of this lack of language was, in her view, that her pupils could not look at a problem and know which strategy to use. The other teacher said that her class knew about making tables and looking for patterns.

Once again there was little or no difference between the responses of the teachers in the two groups, with the predominant response being that few of the classes had any knowledge of strategies prior to the year of the study.

*3. In the course of the year did you mention particular strategies to them? If 'Yes', (a) which ones? (b) when and how was this done?*

### Group A

It is worth recalling that the teachers of the group A classes had been given no specific briefing to emphasise the importance of identifying and reporting of strategies during their problem solving teaching, in the way that the teachers of the group B classes had.

The purpose of this particular question was to ascertain the extent to which the teachers of the group A classes had independently followed the same approach to the teaching and learning of strategies as that which had been suggested to the group B teachers in their initial briefings. The methods used by group A teachers to give their pupils a working knowledge of a range of strategies constituted one of the variables over which the researcher had no control. The responses to this question showed how closely their approaches mirrored those of the group B teachers. If they arrived by whatever routes, at approaches similar to those of the group B teachers, their pupils' knowledge of strategies and their abilities to describe their processes at the end of the study could be expected not to differ significantly from those of their group B counterparts. As far as the researcher was aware, there was no contact between either individual teachers within each group, or between the two groups.

Of the six group A teachers, three said they had made explicit mention of strategies by name and three said that they had not done so. Those who did not make explicit mention of strategies all remarked on the fact that their pupils learned the strategies anyway.

I haven't, no. But because they were doing examples from the one folder, they picked up on the idea and said, "Oh, I know what to do now." So once we'd got to that stage we changed to a different folder and they had to work out how they were going to solve these problems. They said things like, "It's like the last one, you make a list or organise a list," and things like that. They were beginning to talk in problem solving language.

This remark highlights an important development in pupils' problem solving skills, namely an ability to recognise situations similar to ones met previously and to be able to apply the same strategies as were used on earlier occasions.

Another teacher said,

I would choose a problem and do it on the board with two thirds of them. They would say, "With this one we'll have to make a list", or "You just have to guess and try it out".

The three teachers who did make explicit reference to strategies behaved in a way similar to the teachers in group B. One said,

I used all the strategies in the files. The first term we did folders A and B and worked on these strategies, then C and D and E and F. Towards the end I was just giving them odd ones out of the different files to see what they made of them. I would give them the problems first and then we talked about the strategies. We did this in a group. It was very much oral discussion about it. Quite often they didn't have an answer so they were quite willing to discuss various answers and how they got them.

Another, in a similar vein said,

I made a point of mentioning strategies by name. I would give them so many problems and mix them up so that they didn't know, then say, "Can you identify what you were using?" I would put them on the board and they did identify them. I had said we would be using strategies and asked them to identify what they had been using to work that problem out. They had a record sheet which they filled in so that they were putting down what they were actually using.

The remaining teacher in this group said that she referred to the strategies which were highlighted in the commercial text which she used with her class.

The strategies used were, for the most part, all or some of those which were the focus of each of the files of problems which had been given by the researcher to all the teachers participating in the study. Pupils had been encouraged to discuss the strategies used either in groups or with the teacher working out the problem with them on the blackboard. There was one reference to pupils recording their processes on a sheet, which was precisely what the group B teachers had been requested to do with their classes. This teacher had obviously decided that this was a useful technique to help her pupils and is an illustration of the difficulty of controlling the variable of teacher behaviour in a study such as this.

### Group B

Five of the seven group B teachers said that they had tried to make their pupils aware of strategies in an explicit way. The other two were like the teacher quoted above who hoped that their pupils would pick them up from the repeated exposure to problems from the same folder -

I tried not to push them too much in one direction. I tried very much to see if they could come up with the goods themselves.

Most of the teachers in this group made use of class or group discussion to agree on or suggest possible strategies and there was widespread use of charts on the wall to which pupils could refer for help when deciding on a strategy to use, or when trying to recognise, identify and name the one which had been used to solve a problem.

The picture emerging from both groups of responses is one of much discussion and collective decision-making to focus on the strategies used. Teachers in group B seem to have been more explicit in their attempts to get their pupils to think about and identify strategies than those in group A. Those in group A, however, seem to have succeeded in making their pupils aware of strategies through a slightly less direct use of explicit references to them, relying instead on the pupils themselves to recognise strategies from their repeated experiences of them in the sets of problems in the folders.

*4. Are most of your pupils aware of some problem solving strategies now? If 'Yes' (a) which strategies? (b) how do you know they are?*

#### Group A

All thirteen teachers in both groups answered this question in the affirmative and referred to the strategies in their folders. Some of them had not given their pupils all the strategies but had restricted them to a subset of those in the folders. Three of the teachers noted that they saw their pupils using strategies. Another two referred to the fact that some discussion took place about which strategies should be used -

I would give them out and say, "Look at it for five minutes. Think about how you are going to solve it, what strategies to use," etc. Then I would go round the groups and say, "Right, what are we going to use here?" I was building the strategies into the discussion.

Another teacher referred to the way her pupils had begun to talk in problem solving language and how their use of strategies was becoming more refined. They not only knew, for example, that they should make a list, but also that they should try to do so in an organised way.

## Group B

This group also made reference to the discussion which took place with children being encouraged to talk in terms of strategies. The list of strategies on the classroom wall and the use of the recording sheets were also referred to.

Throughout the year we were dipping into the folders for the problem solving and the children filled in a checklist. One of the columns said "Which strategies did you employ?" and again we kept coming back to the chart. I did make it clear that there are far more strategies to adopt, but we kept homing in on the chart.

One teacher reported on a visit to her school by the HMII and said how pleased they had been with the children's problem solving.

They (the inspectors) were very impressed. That was in January. They were very pleased that they (the pupils) were able to speak about their problem solving, knew how to do it and the method they were using and also the least help that they were given by both teachers, not just myself.

There was general agreement that most of the pupils were aware of a number of strategies. They were willing to express opinions about which to use and, although they did not always make an appropriate choice on the first occasion, they now had enough knowledge to be able to try an alternative approach if the first one did not seem to be leading anywhere. This feeling was shared between both groups of teachers.

*5. What other skills or attitudes with respect to problem solving do you feel your pupils have acquired this year?*

## Group A

This question elicited a number of lengthy and enthusiastic responses from teachers. Frequent references were made to the fact that the pupils had much more positive attitudes at the end of the year. They had lost their fear and dislike of problems and were more willing to tackle them. Much more open discussion took place and the children seemed not only more willing to cooperate with their colleagues but were also better at working together. Typical of the views expressed are the following extracts from responses.

Their attitude is now more positive. They won't sit down and let it beat them. They now have something to work with and they are more enthusiastic. They enjoy the day when we do problem solving. It's not something they are frightened of. That was picked up in the HMI report as well - that they had an open attitude and they were enjoying problem solving. The skills they use are developing over the course of the year. They develop these skills for themselves. I didn't say to them, "This is how we're doing these sets of questions." They found them out and they apply them now quite openly to other problems. They had done some problem solving previously but it was very haphazard and unstructured.

and

They are more willing to tackle a problem and they don't give up so easily now. They used to come out after ten minutes saying they didn't understand it. They don't do that now. They work away at it. They take pleasure and become quite excited now when they figure it out. If they hear someone else say they are stuck they will help each other. I could see their attitude changing. At first they would say, "Oh no, problems," when they saw them on the board, but now they say to me, "I've finished. Will I do a problem?" So they have gained in confidence.

### Group B

The teachers in this group were equally enthusiastic and effusive about their pupils' gains from their problem solving programme and raised a few points in addition to those made by teachers in group A. Like their colleagues in group A schools, they referred to the pupils' willingness to look for a strategy, to persevere and show patience without giving up and to realise the need to spend time looking for and working towards a solution.

They are not thinking, "I have to get this answer in ten seconds." They realise it is going to take maybe twenty or thirty minutes to work it out and that if that doesn't work they have to try something else. They don't discard things quite so readily.

Another benefit noted by teachers in this group was the fact that their pupils now realised that their time had not been wasted if they did not achieve the correct answer. They now appreciated that trying and having a go was in itself a worthwhile activity and this belief was being reflected in other areas of their work. This transfer of skills and attitudes had not been a goal of the study but had been a welcome side effect. Another such transferred skill was noted by a teacher who said,

They are really very enthusiastic about it and it is not only with the problem solving sheets. If I give them a problem to solve outwith the problem solving sheets

they are quite positive about it and enthusiastic. They also read their own maths better.

Two teachers noted that their pupils were more selective and discriminating about what they wrote down and about how they set it down. They contrasted the fact that at the beginning of the year most pupils would write down far too much, in some cases almost re-writing the problem. At the end of the year they tended to focus on the relevant information and try to use it in a more economical way.

They now think more about the process than the end result. They're thinking more about how they're setting it down and are trying to be more logical, whereas before they just put all their thoughts down and there was no sense of how they wrote it down.

The responses given in this section from both groups of teachers were, in the researcher's view, extremely encouraging in the sense that they identified a number of intended outcomes but that they also produced evidence of some outcomes which were less expected. These were the skills and attitudes which the pupils showed were being transferred to areas other than problem solving.

#### *6. Did your pupils 'report' on their problem solving activities?*

Teachers were asked to select one of the categories 'Always', 'Usually', 'Sometimes' or 'Never'. All of the group A teachers and all but two of those in group B chose the 'Sometimes' response. The other two said that their pupils were 'Always' asked to report. These results were at odds with the responses given in Table 6.29 to the same question asked of pupils (question 8, Section 6.5), in which all of the group B ones said they 'Always' had to report and five group A pupils saying that they did not have to report. It is difficult to explain this discrepancy, other than to suggest differing perceptions of the words 'always' and 'sometimes'.



*6(a). How was this reporting done?*

Group A

Teachers in group A had been given no explicit advice on reporting by the researcher in the initial briefing sessions. Five out of the six teachers said that their pupils reported orally to them, either at the end of the session when she asked them how they had done the problem or as she went round the class visiting different groups. One teacher made a practice of having a pupil explain to the class how he or she had solved that particular day's problem. Only one of the teachers in this group asked their pupils to produce a written report on their record sheets. She also used a number of other ways, mindful perhaps of the advice in the Guidelines that reporting should be done, "Using a variety of ways". (SOED, 1991, p12).

Group B

Six out of the seven teachers in this group referred to the use of the sheet which the researcher had requested that they use, as part of the process which was unique to their group. The seventh did not specify how her class reported. In addition to using the sheets mentioned (and shown in Appendix 4.7) all the teachers in this group also regularly asked their pupils to report orally, either individually to the teacher or in a whole class setting.

They nearly always reported on their activities, sometimes more fully than at others, using the record sheets that they had. We also have a plenary session after the problem solving, to discuss different methods and just to talk them through. We kept the checklist very short and concise. I think it was perhaps adapted from the one you suggested. It was really very simple - date, the problem and how did you solve it - very simple. In the verbal reports through discussion they opened up a little more. There wasn't always time to do that, I must confess. Sometimes it was done very quickly, but the verbal reports were better. They opened up discussion.

There was an obvious difference between the responses of the two groups to this question, clearly resulting from the different requests which had been made to the two groups in the initial briefing sessions given by the researcher. All of the group B teachers, with one possible exception, had complied with the request to use the recording sheet with the additional column in which the pupils had to describe their solution process. All

of them, however, had unilaterally decided to complement this form of reporting with frequent oral reports, in the belief that these allowed for and encouraged a greater openness in description and discussion of the processes. The effects of the practices of both sets of teachers were that, as had been intended by the researcher at the beginning of the study, pupils in the group B classes had been given more frequent and varied opportunities to report on their problem solving strategies than had those in the group A classes.

*6(b). Did they find it difficult to report?*

Groups A and B

The responses of both sets of teachers to this question were similar. They all said that their pupils had found it difficult at the beginning of the year. Typical comments were,

They are not so bad now but they definitely found it difficult to begin with. They were not sure how much to put down. If they got their way through it they didn't think they had to report it at all.

and

They found it difficult and would sometimes say, "I just did it," or "I knew how to do it."

These responses, referring to the difficulties children had in describing their processes, had been predicted by the researcher based on his own previous work with problem solving (Logan, 1996) and on the work of others previously referred to in Section 2.2.6. The following question was one which would perhaps convey more useful data.

*6(c). Did their ability to 'report' improve as the year progressed?*

Group A

All the teachers in this group felt that their pupils had improved as the year went on.

They became more confident that there were things they could apply and strategies they could try out. They became a bit more relaxed about it all.

## Group B

These teachers too felt that they had seen an improvement in their pupils' ability to report. There were references to more organisation and structure in the pupils' reports with more frequent mention of strategies.

They definitely improved in their ability to report. In the beginning they hadn't a clue. Now they use strategy words using the lists of strategies on the wall.

The results coming from both groups confirm that the children's ability to describe their problem solving processes and to refer to a number of commonly used problem solving strategies had improved by the end of the year's programme.

### *7. How did you assess your pupils' problem solving activities?*

## Group A

Four of the teachers in this group said that they did not assess their pupils and the other two said that they assessed by observation and discussion. From the comments made in response to the question it seems clear that in fact most of the teachers did do some informal observational assessment and that those who said they did not assess probably meant that they did not do any formal or written assessment. Most teachers seemed to agree with the comment made by one that,

I had enough evidence from watching them to enable me to complete the reporting sheet. I did get to know who was becoming more confident and competent.

Two teachers expressed the view that assessment was one of the things that they would now have to build into their problem solving programme.

In their initial briefing sessions with the researcher, the advice given to the teachers had been to keep assessment informal and observational. This was a conscious decision to avoid the possibility of the whole programme being unduly influenced by assessment considerations. The minimal requirement was agreed to be the need to be able to provide parents with a brief sentence or two about their child's progress in problem solving over

the year. This was not at odds with the *Guidelines* which do not specify any levels or benchmarks against which pupils' problem solving performance could be judged, so it seemed that the most pragmatic way to proceed was the way that had been suggested. This is what most teachers in both groups seem to have done and the majority appear to be happy with this process.

### Group B

The views expressed by the teachers in this group were once again very much in line with those of group A. There were two teachers however, who clearly felt the need to be more detailed in their assessment procedures. They both kept a record for assessment purposes for each child on each problem, for example by indicating with a tick whether they, 'solved it', 'solved it with help' or 'found strategies but never solved it'.

### *8. How did you keep a record of their problem solving activities?*

### Group A

All the teachers reported they had used the recording sheets provided for them by the researcher and which appear in Appendix 4.2. In fact it is clear from the responses from two of the teachers in the group that they did not use this sheet (although they said they had used it) but used instead a version of it similar to the one given to the group B classes. This latter version (Appendix 4.7) invited the pupils to describe how they had solved each problem. It is not known whether these two teachers acted independently to amend the sheet they were given, or whether they were given sight of the group B recording sheet and decided to use it in preference to their own one. However they came by it, it seems to have been well received.

The children used the record sheets that you provided. In fact they quite enjoyed being responsible for recording when and what and how they'd actually tried to solve the problem.

In the cases of the other four classes the pupils themselves kept the record sheet up to date. In one case the teacher used the same sheet but did the record keeping herself instead of letting the pupils do so, commenting that she found the sheet helpful.

### Group B

All of the pupils in the group B classes kept their own records using the sheets provided. There was no evidence that their teachers kept records of their own. One or two of the teachers had also picked up the idea of using a wall grid to show at a glance the number and spread of the problems over various strategies which had been done by each child. Examples of such grids had also been shown to the teachers during their initial briefing. One teacher, explaining why these had been found helpful said,

The children keep a record of their own. We also have a wall chart and the children had great fun colouring in all the boxes as to what they had been up to etc. It also gave me an idea who wasn't keen to work on certain areas. It becomes obvious the type of problems that a particular duo or person is not coping with.

There was a clear consensus among both groups of teachers that the pupils' self-recording sheets had been seen as a success. They gave the pupils the responsibility for keeping and maintaining their own records whilst freeing the teachers from an additional record-keeping task. The minority of teachers for whom this was an inadequate form of recording were of course, free to augment their record-keeping procedures in some of the ways described.

*9. Were you happy with the organisation and amount of your problem solving resources?*

*(a) If not, how would you change them?*

### Group A

The organisation and resources referred to in the question were described in Section 4.2.5. The resources consisted of six folders, each one representing a different problem solving strategy and containing problems appropriate to three levels of attainment. The organisation and use of the resources were suggested to the teachers in the initial briefing session.

All teachers agreed that there were sufficient resources, more in fact than they could get through in a year. Two teachers felt that they would adjust the way they used them in future.

I think what I would do next time would be to teach the strategies by doing two or three examples of each and then allow the children to select from a mixture of strategies so that once they'd got the hang of it they could then apply them. I was very happy to have your structure to start with.

Other teachers were happy with things the way they were.

I don't think I would change anything because I think it's an area where they need to feel they are working as a whole group and the poorer ones are supported by the better ones. .... No there didn't seem to be a problem with the brighter ones getting the answers first. I think they all enjoyed saying, "Well, I managed this," and it was helpful for the poorer ones being brought up a bit.

Two other points were made by teachers in response to this question. The first was they would have appreciated more problems at a lower level, since they had pupils in their classes operating at levels B, C and D. This is a reasonable request, but the initial description of the materials was that they would be suitable for P7 classes working at or towards level D. The other request was for answers to the problems.

The answers would have been a help. Sometimes I put one on the board without knowing the answer in the hope that one of the children would come up with the answer.

This request for answers was made on several occasions by teachers involved in the study, but was resisted by the researcher for two reasons. It was hoped that the pupils might get a boost from the feeling that their teachers did not know the answers and that this might generate a feeling of all being in it together. Secondly, the pressure of time was such that the researcher did not have time to produce a set of answers before the study got under way. In the event, a set of answers was produced and circulated to all participating teachers at the end of the year. Solution details, in terms of processes or strategies, were not provided, since it had been made clear to the teachers in their initial briefings that most of the problems in their folders could be solved in a number of ways, using different strategies.

## Group B

The views of this group were very similar to those expressed by the group A teachers. On the theme of there being no answers, one teacher remarked that,

The children just assumed that I knew the answers and it actually did them some good to see that problem solving wasn't just for them, it was for everyone and it goes on all through your life. So we solved some of the problems together.

This was precisely the kind of outcome the researcher had been hoping for in taking the decision not to issue answers at the outset. It should be said, however, that this was not the view taken by most teachers.

There were a few reservations again from this group about the lack of resources suitable for the lower attaining children in the class. Such remarks tended to come from teachers who had composite classes covering three or sometimes four primary stages and as a consequence including pupils working at most levels from A to E. Provision of materials to cope with such a range of attainment was beyond the scope of this project.

The final comment in this section related to the time teachers had allocated to problem solving. Some teachers commented on the fact that pupils became much quicker as the year went on. One teacher remarked that it had been much more time consuming than she had imagined it would be and admitted that her time management of problem solving had not worked out as she had hoped it would.

Very often we would do perhaps three in one day and it became a problem solving maths day rather than embedding it in the curriculum as I hoped at the beginning of the year. I think I could do better next time.

These responses represented for the researcher encouraging signs that the teachers involved in the study were now displaying sufficient confidence in their own ability to begin to personalise their problem solving programmes to fit in with their own views, beliefs and approaches to teaching. A number of them were now prepared to take the

materials as originally presented and modify, adjust and add to them to provide an even better experience for their own classes.

*10. How do you feel about the success or failure of the problem solving programme you have followed this year?*

### Group A

All the teachers felt that the programme had been successful, although one said that she felt she could do better. The following is a typical comment.

I have covered problem solving in a much more organised way than ever before and pupils are much more capable of solving problems now. I would say this is the first year I feel very satisfied with problem solving. That was why I volunteered as this was an area we didn't cover adequately. This will be a basis for my own practice in future.

Other teachers commented on how much the children had got out of the programme.

### Group B

These teachers also enjoyed the programme with only one or two expressing minor reservations. The structure and organisation of the programme were generally appreciated and some teachers felt more secure about problem solving now that they had a structure which made it easier for them to do it on a regular basis.

I liked the programme because of the structure of it. I liked it because it was there at hand and I liked the structural formation. The children liked it because they thought they were getting freedom of choice. The self-recording and working together were positive things.

One teacher remarked that it was almost like "a computer game on paper to a lot of the children". There was also a repeated request for more materials for the lower end of the attainment range and one teacher commented on the unreliability, as she saw it, of the classification of the problems into levels of difficulty using circles, triangles and squares.



The difficulty of getting pupils to explain how they had solved the problems was discussed by one teacher at some length. When referring to her efforts at encouraging children to verbalise, she remarked,

I have always found that extremely difficult. Hopefully this year they have found it a little bit easier to put into words. I have two or three girls who virtually write a story to get to the answer. You can see the strategies and the working out but they don't have the technique of how to separate all the bits.

There were few noticeable differences between the two groups in their responses to this question.

*11. What would you do differently if you were to follow a similar programme again?*

Group A

A number of ideas were mentioned here. One teacher would do it in bigger blocks of time, but only once every four weeks. Another would have her pupils work in pairs. Two teachers said that they would do the same again but with much more confidence.

I'd be more confident myself in teaching strategies now, rather than saying, "Oh, I must do some problem solving," and throwing something that looks like a problem at them, which to be honest is what I was doing before. I'd like to thank you very much for allowing me to be involved, for it's taught me so much as well.

Two teachers would change their record keeping, one to keep a better personal record of the children's problem solving performance, the other to keep a wall chart to let everyone see at a glance who has done what. Other suggested changes related to the timing of the introduction of different strategies and the setting up of a bank of solutions for the problems in the folders.

Group B

These teachers also had a number of proposals for minor changes. Some of these were concerned with the way classes had been organised. One teacher, for example, intended to abandon the practice of having pupils work in fixed pairings in favour of having them work in larger groups. Another was going to have whole groups in her class work on the same problem, though not necessarily as a group. This was for ease of monitoring and

assessing. More practical problem solving work, greater integration of problem solving into the regular mathematics curriculum and the deployment of additional materials were also mooted.

## *12. Any other comments?*

### Groups A and B

All of the comments received in response to this question were about how much the teachers had benefitted from the experience, as illustrated by the following comments.

I quite enjoyed the challenge. The organisation is hard work but it's a worthwhile exercise.

I asked them which ones they found hard and they told me that they found 'work backwards' and 'make a list' easy. They liked those ones and they liked 'logical reasoning'. Those who liked the 'logical reasoning' ones were generally the brighter ones.

I was glad of the extra resources supplied. At the beginning of the year they said, "Oh, don't give us the second test, that first one was too hard," and at the end of the year their response was, "Oh yes, that was great. It only took a minute. Can we get the second test now?"

It has been a very good experience. I think they got a lot out of it. If anything, it has saved me time and worry about getting through problem solving for P7 this year and it really was excellent and I found the structure very good.

I have enjoyed it and I think the children enjoyed it too.

## **6.7 Conclusions**

From the teachers' point of view, the programme seems to have been both a valuable learning experience and a worthwhile exercise in professional development, and one that they all claim to have enjoyed.

In their year-end interviews they were extremely positive with only one or two minor difficulties identified. The most common of these related to the need expressed by teachers on a number of occasions for more materials for their lower attaining pupils - those operating at the 5-14 levels B and C. Some teachers also felt that they would like to

make adjustments to the way their pupils' work was recorded, but seemed to be confident that they could undertake these changes themselves.

The reactions of the pupils involved in the study were equally positive. For pupils other than those who were the younger ones in composite classes or who were the lowest attainers in P6 and P7, the results were almost universally encouraging. At the end of the year they had more confidence, much more positive attitudes and had a wider range and knowledge of ideas to help get them started with problems. Words like 'patience', 'tenacity', and 'determination' were used freely by teachers to describe the attributes acquired by their pupils in the course of the year's programme. The pupils also seemed to have acquired a greater realisation of the importance of processes in problem solving. Pupils had also become better at working cooperatively and it was encouraging to learn that many of these advantages gained by pupils in the affective domain were being transferred to other areas of their learning.

In terms of the study and the comparison between pupils in the two groups, the data gathered from these interviews confirmed that the teachers in group B had, in general, implemented the request made of them by the researcher to focus on strategies by emphasising the reporting aspect of problem solving. It also transpired though that two of the group A teachers had behaved in a way similar to that requested of the group B teachers, with a possible distorting effect on the data obtained. In fact, as was pointed out at the end of section 6.1, no statistically significant changes in the group A/group B comparisons resulted from classifying these two teachers as group B.

As noted earlier in the chapter, there was no significant difference in the performances of the two groups in problem solving over the year. There were occasions when one group performed slightly better than the other, but the quantitative data collected showed that when this happened the differences were not significant.

There was more evidence of differences between the groups when the pupil interview data were analysed and discussed. The group B pupils seemed to be showing evidence of more metacognitive thinking than their group A counterparts in the responses they gave to the researcher's interventions and questions. As was noted earlier, however, these results should be treated with some caution because of the possible lack of reliability of the coding system used to try to quantify the verbal responses given by the pupils. The analysis and categorisation of pupil responses was based on a scrutiny of the transcripts of the interviews and observations and was done by the researcher working as an individual. This should be recognised when considering the evidence which apparently showed, for example, group B's greater improvement in metacognitive performance from term 1 to term 2 (Tables 6.20 and 6.22).

The qualitative data in Section 6.5, gathered from pupils' year-end interviews, provided evidence that the extra teaching given to group B pupils had resulted in their having a greater knowledge of strategies than their group A counterparts. On the basis of this evidence, the X<sub>2</sub> treatment as described in Figure 4.2 in Section 4.2.7 which was given exclusively to group B pupils, seems to have been effective. It was pleasing, however, to note that the X<sub>1</sub> treatment which both groups experienced, in the form of the planned and structured problem solving programme, had produced very positive results for both groups of pupils.

In conclusion, while there was no evidence of one group's problem solving performance having improved more than the other's in terms of the scores produced, there is enough evidence to conclude that group B's knowledge of strategies was enhanced to a greater extent than group A's. The most gratifying result, however, was that both groups of pupils seemed to have produced very positive results in the cognitive, the metacognitive and the affective domains.

## Chapter 7 Conclusions

### 7.1 Introduction

This chapter will summarise the main findings and conclusions of both parts of this research. Sections 7.2 and 7.3 will address the results of the national survey in relation to Research Questions 1 and 2. Section 7.4 will evaluate the national survey.

Sections 7.5 - 7.11 will discuss the findings of the in-depth problem solving study in relation to Research Questions 3 - 6 and section 7.12 will evaluate the results of this study.

### 7.2 Research question 1

*What are primary teachers' views on the content and implementation of the 5-14 National Guidelines in Mathematics?*

The most salient feature of the results of the national survey was the generally widespread acceptance of the *Guidelines*. In spite of the fact their implementation demanded a number of changes in the content and methods of the primary mathematics curriculum, they were welcomed by nearly all the teachers in this survey. The Scottish Office Education and Industry Department should be able to derive some satisfaction from teachers' very positive reactions to the mathematics *Guidelines*. As well as being happy with their content and purpose, teachers were appreciative of all the forms of support they had received in their implementation. A disappointingly small number of teachers, however, claim to have seen some of the nationally produced support materials.

Teachers noted that changes had been effected in a number of key areas, including assessment, planning, content taught, methods of teaching and recording. Of these, only

in the area of recording was the change seen as major. The new 5-14 framework of outcomes, strands, targets and levels was welcomed as an aid to planning, assessing and recording. In terms of the new content which teachers felt obliged to teach subsequent to the introduction of the *Guidelines*, the problem solving and enquiry outcome was seen as the most significant addition. Although greater use of practical work was not advocated explicitly in the *Guidelines*, a significant minority of teachers in the survey (21%) specified this as the biggest change in teaching methods. This suggests that teachers were adopting not only the letter, but also the spirit of the *Guidelines*.

The broad acceptance of what were seen as the sensible recommendations made in the *Guidelines* was tempered by teachers' awareness of classroom realities. This led to a number of staff development and resource needs being expressed by teachers in the survey. The one most frequently noted was for more time to adapt to and assimilate the new ideas in the *Guidelines*. Almost half of the teachers in the sample expressed a desire for more time for professional activities such as reading, planning, organising teaching materials and resources, and for discussions with colleagues. It should be noted that these sentiments may not be attributable solely to the implementation of the mathematics *Guidelines*, but rather to the cumulative effect of schools having five sets of guidelines to deal with simultaneously.

In terms of perceived staff development needs, most of these were related to the problem solving and enquiry outcome. Most teachers said they would appreciate advice on levels, progression and differentiation within this outcome. There was also an expression of need for some professional development with computers and computer software, with most teachers dissatisfied with the small amount of time spent by their pupils using computers for mathematics in the classroom.

In these respects teachers in Scotland differed from many of their colleagues in England and Wales, who had expressed concern with aspects of the content of the mathematics

curriculum, with management and organisational issues and with class size, none of which were mentioned in this survey.

### 7.3 Research question 2

*Which of the views expressed by teachers in this survey should inform future curriculum innovations?*

Although the *Guidelines* have been seen as an overwhelmingly well-received curricular innovation, there are some aspects of their implementation which, viewed in retrospect, could have made them even more successful.

The main lesson for policy makers from the results of this survey is that teachers needed more time to implement curricular innovations such as this. Teachers' acceptance of the contents and the message of the *Guidelines*, as noted above, was tempered by the widely expressed desire to be able to conduct the implementation at a more relaxed pace, thereby allowing them to reflect on and come to terms with the new ideas being promoted. Policy makers at the national level, many of whom have been secondary school single subject specialist teachers, should remember that primary teachers have responsibility for every one of the subject areas for which guidelines were produced. Whilst the support provided nationally was appreciated, it was still considered inadequate and had not even been seen by most teachers.

The forms of support most widely available and enjoyed were provided locally by regional education authorities. These local authorities each had to conduct their own needs' analyses and make decisions about staff development activities to be delivered locally. The only nationally coordinated event to support local authorities in the implementation of the *Guidelines* took the form of a single one-day conference to highlight the issues most likely to feature in future staff development needs. Whilst this conference was appreciated and clearly went some way to meeting a need, similar events

at regular intervals, to provide some central coordination and advice about the direction and nature of the kinds of activities needed to help in the successful implementation of the *Guidelines*, would have saved much duplication of effort throughout the country. This gap between national and regional provision was not commented on in this survey by teachers, but was noted by the researcher in his capacity as National Development Officer for 5-14 Mathematics, and was, in his view, a major factor contributing to the need for more support expressed by many teachers.

The inclusion of problem solving and enquiry as an outcome to be taught, needed to be accompanied by a greater range of support mechanisms and staff development initiatives to help answer some of the many queries teachers had about it. The exemplar materials which were published nationally by the Scottish Consultative Council on the Curriculum (SCCC, 1993a, 1993b) were well received by the relatively small number of teachers who saw them. Many more such materials could profitably have been produced over a longer period of time, to help teachers resolve some of the difficult issues related to planning and teaching problem solving referred to above. Such materials alone would have to be accompanied by mechanisms for ensuring that they would be accessible to teachers.

Many teachers feel that they are not fully exploiting the potential of information technology in their mathematics curriculum. The nature of the development of information and communication technology is such that teachers have been faced with many changes in both hardware and software over the past few years and a period of consolidation with one type of computer would be welcomed. Some advice on appropriate software and its relevance to the mathematics *Guidelines* would also be welcomed by teachers. There is also a continuing demand for more computers to allow pupils to spend more time on them.

In summary, more time, in teachers' views, should be allocated for implementation of similar curriculum changes. More support, either provided nationally or supported by nationally produced materials and staff development initiatives, would have been



appreciated by teachers. Much more advice was needed by teachers to help with the implementation of the problem solving and enquiry outcome and there was a widely shared view of the need for a rationalisation of developments in information technology.

## **7.4 An evaluation of the national survey**

### Design and conduct of the survey

The choice of evaluation model used in this survey was influenced by a number of theoretical and practical considerations which were identified in the literature reviewed in Section 2.1.

Some of the historical models discussed in Chapter 2 were not considered for use in this survey. The absence of clearly stated objectives for the *Guidelines* meant that a classical model was inappropriate. Other approaches, such as that advocated by Stake (1967a) in his 'countenance' model and Stufflebeam's (1972) CIPP framework were deemed to be too complex and unsuited to the purposes of this evaluation. The model designed for this survey borrowed heavily from several more qualitative approaches to evaluation and attempted to meet the following criteria, all of which were identified in the literature review in Section 2.1.

The survey was intended to be, in Stake's (1975) terms, 'responsive' in the sense that the researcher was concerned to make the findings interesting and useful to those involved, namely primary teachers in Scotland.

The evaluation was also seen as 'democratic', as defined by MacDonald (1976), inasmuch as it was designed to provide information about teachers' views of the implementation of the *Guidelines* in ways which make the results accessible to a wide audience, many of whom might be non-specialists.

In the absence of a set of highly specified objectives for the *Guidelines*, the evaluation model used had to provide for the emergence of unintended or unexpected outcomes. In this respect it exemplified Scriven's (1967) 'goal-free' approach to evaluation in which the success of the curricular initiative is judged more by its actual achievements than by the extent to which it attained any pre-stated intentions or goals. In this case the goals had been stated only in very general terms as 'needs for action' (Section 1.1, pp1-2).

The approach to evaluation advocated by Nixon (1992), which argued for increased involvement of teachers in the evaluation of any curricular initiative, was reflected in the design of this study. It also incorporated aspects of Parlett and Hamilton's (1976) 'illuminative' approach with its emphasis on reporting how the successes and shortcomings of this particular curricular innovation were viewed by those 'directly concerned', i.e. the teachers in the survey.

The selection of these criteria in the design of this evaluation model suggests an approach which was primarily qualitative in nature, but which sought to reflect concerns expressed by Stenhouse (1979) by including a 'quantitative ingredient'. The outcomes emerging from the study, which are portrayed in the following pages, reflect the influence of the literature review on the design of the theoretical framework chosen and would appear to justify the description of it as one which owes no allegiance to any one particular theoretical model but which contains elements of illuminative, portrayal, democratic and responsive antecedents.

The researcher was, in general, satisfied that the survey had been successful in producing the kind of information sought. The decision to restrict the survey to one nationally distributed questionnaire was based on pragmatic considerations. It would have been instructive to have supplemented the data by conducting a number of randomly arranged interviews and to have carried out some triangulation by surveying the views of pupils and head teachers. Factors of time and cost prevented these from happening. None of the responding teachers commented unfavourably on the design of the questionnaire and the

acceptable response rate of 36% of teachers contacted, at a time of great pressure on schools, was interpreted as a sign that teachers were willing to engage with the issues raised in the questionnaire.

The questionnaire and interviews which made up the pilot study seemed to have been effective in identifying the main issues to be explored in the full survey. None of the teachers responding to the questionnaire raised issues different from those chosen by the researcher for inclusion in it.

The conduct of the survey ran smoothly, and full cooperation was given by all the local education authorities in the country, when they were approached seeking permission to conduct the survey in their schools. The information supplied to the researcher by the Scottish Examination Board and the Scottish Council for Research in Education was helpful in avoiding schools involved in other recent surveys.

#### Recommendations for future research

The survey suggested two related areas in which future research is needed. The fact that this survey was restricted to primary schools meant that teachers of mathematics in secondary schools did not have the opportunity to respond to the same set of questions. It is, therefore, not known whether they would share some of the concerns or satisfactions expressed by their primary colleagues. It would be interesting to know whether they have identified any post-5-14 differences in knowledge, understanding or attitudes in their incoming S1 pupils, in for example, the area of problem solving and enquiry.

The effects of the *Guidelines* on the transition from primary to secondary schools is another area where some investigation would seem to be needed. Anecdotal evidence, as well as that contained in a recent report by HM Inspectorate (SOEID, 1997), suggests that secondary schools have not been using pupils' primary school records of attainment, which refer to the 5-14 levels A - E, to provide the degree of curricular continuity envisaged when all sets of national guidelines were first conceived. At a time when there

is a push towards 'setting' in S1 (SOEID, 1997), it would seem sensible to expect that assessment results which used the 5-14 levels would provide valuable information to make secondary schools' tasks of establishing 'sets' easier.

### Contributions to the body of knowledge

This survey established the fact that most teachers in Scotland were very receptive to the kind of message being given by the *Guidelines*. Any reservations expressed, tended to be about matters relating to the implementation of the *Guidelines*, rather than their content. In this sense they seem to have been accepted as a worthwhile curriculum initiative by teachers. Differences in this regard were noted between the reactions of Scottish teachers to the mathematics *Guidelines* and those of their counterparts in England and Wales who expressed some misgivings about a number of aspects of the contents of the National Curriculum Mathematics.

Some of the messages contained in the *Guidelines* have been found in this survey to have been inadequately emphasised or clarified. One of the two most significant examples of this concerns the important recommendation about developing pupils' mental calculating strategies. The results of the survey indicate that this message had not been taken on board by most teachers. The other lack of clarification was in the admitted absence of concrete advice given in the *Guidelines* on the teaching of the new problem solving and enquiry outcome. Both of these relative failures, have been, in the researcher's opinion, tacitly admitted by HM Inspectorate. With respect to problem solving, many individual schools have been taken to task in Inspectorate reports, for not demonstrating that they have a coherent, progressive and structured programme of problem solving in place. In *Improving Mathematics Education 5-14* (SOEID, 1997) the message about the importance of mental calculation is re-stated much more vigorously than it was in the *Guidelines*.

Another significant fact to emerge from this survey relates to what has already been stated on several occasions in this report. That is, that teachers were given an inadequate

timescale in which to implement the *Guidelines*. Given the positive ways in which they responded to some of the quite new and demanding ideas of the *Guidelines*, teachers may justifiably have felt that they were entitled to expect more support from central sources for their implementation than they in fact received.

A final observation on the results of this survey relates to some marked differences from similar surveys carried out in relation to the implementation of the National Curriculum Mathematics in England and Wales. Among the most noticeable of these was, firstly, the absence of comment by Scottish teachers about class size as an inhibiting factor in the implementation of the *Guidelines*, and secondly, the fact that teachers in Scotland did not seem, on the whole, to be worried about changes in content to be taught.

### Implications of the survey

At the national level, curriculum developers should, in future, allow more time for teachers to implement new curricular initiatives. They should also provide more central support to help teachers come to terms with aspects of curricular initiatives which are new. This support could take the form of printed materials, courses and conferences for teachers and advisers and the appointment of national development officers to work with local authorities whilst the new developments are being implemented.

At the local level, education authorities should continue to provide support in those areas identified in this survey as causing concern to teachers. These include problem solving, developing mental calculation skills and information and communication technology.

Institutions responsible for initial teacher education should also be aware of these same areas of concern and should ensure that their pre-service and in-service courses address them.

## Conclusion

The overall picture provided by the results of this survey, is of a teaching body committed to trying to implement the national *Guidelines* in a professional way. Whilst, in teachers' perceptions, there were some imperfections and inadequacies in this new curricular initiative, and while more time and resources would have been appreciated, most teachers were demonstrating a willingness to engage with its new approaches and the ideas contained within it.

### **7.5 Research question 3(a)**

*What were the benefits of the year-long structured programme in mathematical problem solving as perceived by the teachers of the P7 classes involved in it?*

The teachers' reactions to their experiences with this problem solving programme were almost universally positive and a number of benefits for both teachers and pupils were identified.

They were very appreciative of the organisational model provided for them at the outset and had enjoyed the security and the support that this structure had given them. A result of this was they now felt more confident and secure in their own ability to plan and teach a problem solving programme. Their growing maturity as teachers of problem solving was demonstrated by the fact that some of them indicated their intention to adjust and personalise the programme for subsequent years to take account of their own particular circumstances.

The greatest number of benefits identified, related to the progress they had noted in their pupils' problem solving skills and attitudes. Amongst the more positive attitudes shown by their pupils were much greater confidence about problem solving and a loss of fear and dislike of this aspect of mathematics. Pupils were better at getting started, had a much greater knowledge of strategies and consequently had more alternative approaches at their

disposal. Teachers noted, too, that whilst their pupils still did not always meet with success, they did not now give up, were prepared to exercise more patience and to persevere with different approaches, in the knowledge that problems often were not solved on the first or second attempt.

By the end of the year pupils were generally better at working together and were willing to discuss their processes with their peers. This discussion had contributed to an awareness of the importance of processes in problem solving and to an improvement in pupils' ability to report on and describe their own strategies and solutions. Related to this growing ability to report, teachers remarked on the success of the self-recording and reporting format which they had been given to use in the study.

An encouraging result of the year's programme, noted by some of the teachers, was the fact that many of the skills and attitudes acquired in this exercise and noted above, were being transferred to other areas of the pupils' work. On the basis of this result alone, the study could be deemed worthwhile.

From the teachers' point of view, the programme seems to have been both a valuable learning experience and a worthwhile exercise in professional development, and one that they all claim to have enjoyed.

## **7.6 Research question 3(b)**

*What were the benefits of the year-long structured programme in mathematical problem solving as perceived by the pupils of the P7 classes involved in it?*

It was clear that most of the pupils involved in the study had achieved benefits in both the cognitive and the affective domains. They all claimed to have learned about a number of problem solving strategies and all agreed that they were more confident about problem solving than they had been at the beginning of the year.

They now appreciated the need for effort, patience and perseverance and they understood that being stuck was part of the problem solving process. Related to the expectation of being stuck was an accompanying awareness of the frequent need to pause, reflect and re-read the question before pursuing an alternative course of action. They had clearly gone beyond the stage of experiencing initial emotional stress commonly felt by children when confronted with a mathematical problem (McLeod 1989).

The growing confidence expressed by all pupils in the study was encouraging to the researcher, and it was to be hoped that this confidence would, as suggested by Meyer and Fenema (1988), have a positive effect on their abilities to solve problems.

## **7.7 Research question 4(a)**

*To what extent does frequent practice in 'reporting' help pupils' metacognitive abilities?*

There is evidence from the interviews conducted with pupils at the end of the study that most of them showed greater use of metacognitive skills than they had earlier in the year. This can be inferred from their explanations of what they did when they were stuck and how, to enable them to get out of the state of being stuck, they were able to select from a bank of strategies which they had learned. If the beliefs of the 'impasse-based' theorists referred to in Section 6.5 are accepted, the ability to select an alternative strategy when stuck is evidence of metacognitive thought. There is also evidence in the results of the study that those pupils in group B who had been encouraged throughout the year to report regularly, demonstrated a knowledge of a greater range of strategies than their peers in group A who had not been so encouraged to report. Whilst these data suggest a link between reporting and increased metacognitive ability, it would be incautious to make more definitive conclusions on the basis of this evidence.



There are similar grounds from the data in Section 6.4, collected from pupil interviews and observations, for concluding that group B pupils were demonstrating a significantly greater degree of metacognitive thought in term 2 than they had done in term 1. It was also noted there that group B pupils outperformed their group A counterparts in terms of observed metacognitive actions throughout the year. Taken at face value, this evidence could suggest that the experience of group B pupils was effective in increasing their metacognitive awareness. These conclusions, however, as was noted in Chapter 6, should be treated with some caution because of the subjective categorisation of pupils' responses into the three metacognitive levels described. The researcher was aware that issues of categorisation will often have contestable elements and may be influenced to some extent by factors such as the researcher's beliefs and the research hypothesis being investigated. The coding and categories used in this study, however, were believed to be acceptable given the context within which the researcher was working.

## **7.8 Research question 4(b)**

*To what extent does frequent practice in 'reporting' help pupils' knowledge of problem solving strategies?*

There was evidence from the responses of both teachers and pupils that those classes whose pupils had been reporting regularly, had a greater knowledge of strategies. Although all of the teachers in the study said that their pupils had learned a range of strategies, subsequent scrutiny of their responses revealed that pupils in group A classes had not been introduced to as many strategies as those in group B. When the pupils were asked about strategies which they had met during the year, there was a much more noticeable difference between the responses of the two groups than there had been in the responses of the teachers, with group B pupils identifying a significantly greater range and number of strategies used. This suggested strongly that the extra emphasis on reporting strategies suggested to, and in fact carried out by, the group B teachers, had been effective in creating a greater knowledge of strategies among their pupils.

## 7.9 Research question 4(c)

*To what extent does frequent practice in 'reporting' help pupils' problem solving skills?*

It is reasonable to conclude, as noted above, that frequent practice in reporting, significantly improves pupils' knowledge of problem solving strategies. It would also seem reasonable to speculate that increased knowledge of problem solving strategies would lead to an improvement in pupils' ability to solve problems. There is, however, no evidence in the results of this study to support such a hypothesis, since no significant difference in the problem solving performances of the two groups over the year was recorded. There were occasions when one group performed slightly better than the other, but the quantitative data collected showed that when this happened the differences were not significant. In addition, an analysis of variance showed that pupils' results were related to the schools they attended, but not to their categorisation as group A or B schools.

## 7.10 Research question 5

*To what extent can the 'reporting' aspect be justified as a component of the problem solving framework in the Guidelines?*

The comments on some of the research questions above have identified two major advantages of frequent reporting. These were a greater perceived use of metacognitive skills and a concomitant increase in pupils' knowledge of strategies. On the basis of the results of this study these are the two demonstrable benefits which can be attributed to the regular emphasis on reporting. In the unlikely event that there were no other benefits deriving from the practice of reporting, these alone could justify the inclusion of reporting as an element of the problem solving framework.

It is, however, worth noting that all of the teachers in the study were using the problem solving framework described in the *Guidelines* and of which the components were Starting, Doing, Reporting and Evaluating. It is difficult to believe that teachers would have recorded such positive reactions to the year's problem solving study, with their references to more open discussion, awareness of the importance of processes and pupils' ability to cooperate, if the reporting component were to be removed from the framework. The Scottish problem solving model is the only one identified by the researcher as containing a reporting component and as a consequence its value has not yet been addressed in the literature. This suggests a possible area for further research, to try to establish conclusively the benefits, which many teachers are convinced exist, of asking pupils to, "explain how you did it".

## 7.11 Research question 6

*What effects has the year's structured programme had on pupils' and teachers' beliefs, views and attitudes to problem solving?*

Pupils and teachers commented in very positive terms about the changes in pupils' attitudes to problem solving. The pupils themselves referred to having greater confidence in their problem solving abilities at the end of the year. They felt that they were more assured of their abilities to get started and were not deterred by being stuck. They were also more aware of the need for patience and perseverance than they had been at the beginning of the year. They realised that solutions did not always come easily and that prolonged effort would often reap rewards. Teachers repeated many of these opinions and also noted that their pupils were much quicker at problem solving. Pupils did not regard time spent on trying unsuccessfully to solve a problem as time wasted, and appreciated the value of having tried a number of strategies as being a worthwhile activity in its own right. The teachers also remarked on the fact that many pupils had shown these attitudes in contexts other than mathematical problem solving.

The teachers themselves were also more positive about teaching problem solving. The organisation and structure of the materials and the programme had given them a degree of confidence about their teaching which they previously did not have. Most of them were sufficiently comfortable with problem solving at the end of the year to be beginning to think of ways in which they could amend and adjust the materials to suit their own particular needs and approaches. Some of them claimed to have enjoyed the challenge of the experience and to have benefitted from it as a staff development exercise.

From the responses collected from both teachers and pupils it was clear that both groups had experienced and observed very definite affective gains from the programme.

## **7.12 An evaluation of the in-depth study**

### Design and conduct of the study

The design of the in-depth study was guided and informed in the first instance by some of the literature reviewed in Chapter 2. The researcher's initial interest in investigating problem solving was given an impetus by the statements of distinguished figures in the field such as Silver (1988) and Lester (1994) among others, who expressed the need for much more research into problem solving which could be used to inform teachers' practice.

The nature of the problems used in the study and the interpretation of 'problem solving' chosen for the study were of the kind exemplified in the *Guidelines* and were similar to those agreed and used by Goldin (1982), Charles and Lester (1984b) and Askew and Wiliam (1995). The results of the study should therefore be of relevance and interest to those concerned with mathematics education in a domain much wider than Scotland. The emphasis in this study on the development of children's metacognitive skills was also a reflection of the concerns expressed throughout the international mathematics education community by people such as Schoenfeld (1982), Garofalo and Lester (1985), and more recently Davidson and Sternberg (1998). This study attempted to address the needs

expressed by such commentators for further investigation of the role of metacognition in the teaching and learning of problem solving.

The other key component of problem solving agreed in much of the literature surveyed in Chapter 2 was the importance of an awareness of strategies and this was also developed in this study. Strategies were first referred to by Polya in 1945 and have been emphasised by most problem solving researchers since then. This study continued this emphasis and, in addition, investigated possible links between metacognition and awareness of some common problem solving strategies. As previously noted in Sections 7.7 and 7.8, this study suggests, using 'reporting' as a context, that such a link has been established.

The framework for children's learning of problem solving used in this study was that of the *Guidelines*, since it represented the best vehicle, in the researcher's view, for the study of children's problem solving processes. The choice of this particular framework was, however, arrived at after careful consideration and scrutiny of a number of others in the literature, (Polya, 1945; Mayer, 1987; Resnick; 1988; Schoenfeld 1992). Whilst none of these frameworks were used for the study, elements of them and ideas and experiences gained by their authors did inform some of the design decisions related to the pupil interviews and observations discussed in Sections 4.2.9 and 6.4.

The literature review was also important in influencing the design and conduct of the pupils observation and interview sessions. In selecting pupils whom their teachers felt to be reasonably articulate and in dismissing 'think aloud' forms of verbalisation, the researcher followed the advice of researchers such as Desforges and Bristow (1995) and Lester, Garofalo and Kroll (1989) respectively. The choice and timing of probings and interventions and the concurrent and retrospective verbalising techniques used were influenced by the articles of Smith and Miller (1978), Ericsson and Simon (1980) and Lawson and Rice (1987). These design decisions, influenced as they were by the literature, were successful in eliciting full and thoughtful responses from most of the pupils used in the interview and observation sessions.

The findings of this study confirmed some aspects of the work of other researchers. Lester's (1996, p666) belief, referred to in Section 2.2.4, that teaching about problem solving strategies does not in general help pupils' problem solving abilities, seems to have been confirmed by the results of this study. The present study also showed, in the teacher and pupil year-end interviews, the amount of change in pupils' attitudes and beliefs about problem solving, thereby endorsing and extending the findings of Cobb, Yackel and Wood (1989), whose work highlighted the benefits for the classroom environment of encouraging pupils to verbalise their solution attempts.

There were several aspects of the work done in this study which, with the benefit of hindsight, the researcher would like to have done differently. The first of these relates to the design of the pre- and post-tests. An analysis similar to the item pair analysis described in Section 6.3 should have been conducted before the start of the study. This would have ensured a much better matching of items on the pre- and post-tests. The items used on the two tests could also have been better matched by strategy, so that the researcher could have stated with more confidence that pupils would do corresponding items on the two tests using similar strategies. These two measures would have provided a more rigorous comparison of pupils' performances on the two tests.

A second aspect of the study which the researcher feels could be reconsidered, relates to the techniques used to attempt to assess changes in children's metacognitive skills. The difficulty of gaining access to pupils' metacognitive processes is well established in the literature and this made the researcher aware of the possible limitations of the techniques used in this study. The observation and intervention model used was partially successful in the sense that the pupils were cooperative and tried, when asked, to explain their thinking. The system of coding the responses given by pupils to the researcher's probes was not rigorous and allowed an element of subjectivity to have a possible distorting effect on the codes allocated to the responses. Although the researcher was conscious of this difficulty, it is still not possible to say that the coding was as objective and reliable as

it ideally might have been. Other possibilities had been considered in an effort to increase the reliability of the coding. A more detailed and complicated coding system would, it was felt, have been no more rigorous. Ideally, additional coders could have been used to encode the pupils' responses. This would have allowed a measure of intercoder reliability to be established and in the event that such a measure was below the normally accepted level of 85% (Green and Gilhooly, 1996) the coding could have been altered and refined. This possibility was considered impractical in view of the fact that the researcher was working on this study as an individual. However, in an effort to attain a greater degree of reliability in each of terms 1 and 2, the researcher had conducted a 'blind' coding in which the responses were coded without reference to the pupils' identities or groups. In both terms the initial coding exercises were repeated after an interval of several weeks. In term 1, 89% of the second codings were identical to the first ones and in term 2, 87% were identical. Whilst this measure clearly does not produce the same level of reliability that an 85% agreement between two different coders would have done, it did suggest that there was a reasonable degree of consistency in the researcher's own coding. The possibility of recording the observations and interviews on videotape for more detailed subsequent scrutiny, possibly involving additional observers, as discussed in Section 2.2.7, was briefly considered but also had to be discounted on grounds of feasibility.

Another difficulty in this study arose in connection with the choice of the two groups of schools. One set of teachers was asked to carry out a particular course of action which was different from the other group. This decision was complicated by the lack of control of the teacher as a variable in both groups. Having made the request to the teachers in group B schools, the researcher had no way of knowing with any certainty, whether the teachers would in fact behave in that way. There was also the possibility that teachers in group A would independently behave in a similar way, which in fact happened in two cases. In addition, the researcher had no knowledge of, or control over, the style and effectiveness of any of the teachers in the study. There was evidence that those teachers who had not been invited to emphasise the 'reporting' component of the problem solving cycle (those in group A), unconsciously compensated by using other effective teaching

measures. It is difficult to think of a way in which teachers' behaviour in this context could have been controlled without being unacceptably intrusive or prescriptive. Perhaps frequent observation and analysis of individual teachers' teaching styles, in advance of the study, might have helped the researcher make a more informed choice of groupings. This would, however, have been impractical for a number of reasons and would certainly have been beyond the scope of this study. Similarly, the possibility of observing the teachers during the study to obtain a 'measure' of their styles, was dismissed as being potentially threatening to them and might have had the effect of fewer teachers volunteering to take part. To study the effectiveness of 'reporting' in improving pupils problem solving skills, it would arguably have been better to have briefed a randomly selected group of teachers, along the lines of the additional briefing given to the group B teachers, and to have compared the problem solving performances of their pupils, with those of a control group of pupils from randomly selected classes.

### Recommendations for future research

The results of this study have suggested a number of benefits deriving from a carefully planned and structured programme of problem solving. It would be interesting to compare the performance of pupils of the teachers in this study with pupils from other randomly selected classes, to determine whether exposure to the experiences of this study, has made the teachers involved more effective in teaching problem solving.

The item pairs analysis technique used here could profitably be developed as a means of predicting 'expected scores' on test items. Using these ideas, some future study could establish norms for the amount by which pupils, after one year, could reasonably be expected to exceed the calculated 'expected score' on selected items. This 'added value' could then be used as a measure against which future innovations in the teaching of problem solving could be judged. Similar techniques could also be used in other areas of mathematics.



Whilst it is hoped that this study will be helpful to teachers at the upper stages in primary schools in planning and teaching problem solving, there are still many questions about problem solving remaining to be answered. Teachers still, eight years after the implementation of the *Guidelines*, have very little knowledge about a hierarchy of problem solving skills and strategies. Many teachers are still, quite reasonably, asking for advice about which problem solving skills or strategies are appropriate for the various stages of the primary school. Further research in this area would help schools to design coherent and progressive programmes of problem solving from P1 to P7.

The benefits of pupil 'reporting' as suggested by the results of this study, related to the development of pupils' knowledge of strategies and there was some evidence to support the hypothesis that pupils' metacognitive skills benefited. There was, however, no evidence that their general problem solving skills were improved by the emphasis on 'reporting'. Further research might help to establish a more conclusive set of links between pupils' ability to describe their processes and the development of their skills in this area.

### Contributions to the body of knowledge

This problem solving study has demonstrated the perceived benefits to teachers and pupils of a carefully structured problem solving programme supported by an organised bank of resources. This programme, along with the advice given to teachers about recording and assessing pupils' problem solving work, seemed to have given teachers a level of security and confidence which they previously did not have. It also generated a set of positive attitudes about problem solving in both teachers and pupils.

Although this study was limited to teachers and pupils at one stage of Scottish primary schools, it is hoped that the findings with respect to the use and application of the resources and organisational methods will be of use to many teachers of other stages on a wider geographical basis. It is also hoped that the message about developing pupils'

knowledge of strategies through teachers' emphasis on 'reporting', will be of benefit to teachers of mathematical problem solving at all levels.

Although this improvement in pupils' working knowledge of strategies was one of the important outcomes noted in this study, there was, however, no evidence of an accompanying improvement in the problem solving skills of the pupils for whom 'reporting' had been encouraged. It could be concluded therefore, that on the basis of this year-long study, 'reporting' of processes does not necessarily help overall problem solving ability.

This study has shown how pupils' attitudes, emotions and beliefs about problem solving can be improved by experiencing a programme such as the one described. The higher than expected gains in problem solving performance over the course of the year, as described in Section 6.3, confirm previous research findings which have established links between positive attitudes and problem solving performance, so it is hoped that the results of this study will also persuade teachers of the benefits of such a programme.

### Implications of the study

The single most significant implication from the results of this study, in the researcher's view, is the need for more research into pupils' acquisition of problem solving skills in mathematics and the role played by metacognition in the development of these skills. In 1991 it was observed in the *Guidelines* that, with respect to problem solving,

At present there is insufficient research evidence or practical experience to define progression and a pragmatic approach is recommended. (SOED 1991, p9)

This study has been an attempt to go some way towards remedying that situation, but there is clearly a continuing need for more research. Such future research, allied to the experiences of teaching problem solving gained by Scottish teachers in the intervening eight years, would help to provide a much more solid base of evidence on which teachers could build future problem solving programmes.

The main implication of this study for teachers, is that they should realise the importance of having a planned, structured and differentiated problem solving programme in place. They should not only be aware of the benefits of including the important element of 'reporting', as advocated in the *Guidelines*, in their problem solving work, but should also be aware that the benefits of 'reporting' do not, on the evidence of this study, necessarily include enhanced problem solving performance. It may, therefore, be non-productive in terms of problem solving skills, to ask pupils to articulate the processes used. It is interesting to speculate whether this observation would still have been made had the pupils involved in this study practised 'reporting' over a longer period of time.

The study has also provided further justification for teaching problem solving, if any were needed, by establishing the fact that some of the important cognitive and affective skills acquired by pupils involved in it were transferred to other areas of their work.

All of these factors, which will be of interest to teachers, should equally inform the work of those involved in the initial education of teachers, so that student teachers, at the beginning of their careers, can begin to engage with issues of how children acquire problem solving skills. It is the researcher's belief that just as pupils' problem solving skills are transferable, so too can the skills of teaching problem solving be usefully transferred to other areas of teachers' work.

In conclusion, it is the researcher's hope that this piece of research, which in Nisbet's (1999) terms can be described as 'practical', will go some way to resolving the difficulties faced by many primary teachers in their attempts to teach an effective programme of mathematical problem solving.

## References

- Adams, H.B. (1986) The teaching of general problem solving strategies. *Gifted Education International*. 4, 85-88.
- Ariav, T. (1986) Curriculum Analysis and Curriculum Evaluation: A Contrast. *Studies in Curriculum Evaluation*, 12, 139-47.
- Andrews, R. (1992) Philosophers of the Classroom. *The Times Educational Supplement*, 3 January, 6.
- Artzt, A.F. and Armour-Thomas, E. (1990) Protocol Analysis of Group Problem Solving in Mathematics: A Cognitive-Metacognitive Framework for Assessment. Paper presented at the Annual Meeting of the American Educational Research Association, Boston.
- Askew, M. and Wiliam, D. (1995) *Recent Research in Mathematics Education* 5-16, OFSTED, London: HMSO.
- Baird, J.R. (1986) Improving learning through enhanced metacognition: a classroom study. *European Journal of Science Education*, 8, 3, 263-282.
- Bennett, S.N., Wragg, E.C., Carré, C.G. and Carter, D.S.G. (1992) A longitudinal study of primary teachers' perceived competence in, and concerns about, National Curriculum implementation. *Research Papers in Education*, 7 (1), 53-78.
- Berl, J., Lewis, G. and Morrison, R. S. (1976) Applying models of choice to the problem of college selection. In J.S.Carroll & J.W.Payne (Eds). *Cognitive and social behaviour*. Hillsdale N.J: Erlbaum.
- Bobbitt, F. (1924) *How to make a Curriculum* Boston: Houghton Mifflin.
- Boud, D. (1989) The role of self-assessment in student grading. *Assessment and Evaluation in Higher Education*. 14, 20-30.
- Boud, D. (1990) Assessment and the promotion of academic values. *Studies in Higher Education*. 15, 101-111.
- Branca, N. and Kilpatrick, J. (1972) The Consistency of Strategies in the Learning of Mathematical Structures. *Journal for Research in Mathematics Education*. 3, 132-140.
- Brown, A.L. (1978) Knowing when, where and how to remember: A problem of metacognition. In R. Glaser (Ed.) *Advances in instructional psychology* (Vol, 1) Hillsdale, N.J: Erlbaum.
- Brown, A. (1987) Metacognition, Executive Control, Self-Regulation and Other More Mysterious Mechanisms. In F.E.Weinert and R.H.Kluwe (Eds) *Metacognition, Motivation and Understanding*. Hillsdale N.J: Erlbaum.
- Brown, C.A., Carpenter, T.P., Kouba, V.L., Lindquist, M.M., Silver, E.A. and Swafford, I.O. (1988) Secondary School results for the Fourth NAEP Mathematics Assessment: Algebra, geometry, mathematical methods and attitudes, *The Mathematics Teacher*, 81, 337-347, 397.
- Brown, M., Johnston, D., Askew, M., Millett, A., Prestage, S. and Walsh, A.S. (1993) *Evaluation of the implementation of the National Curriculum Mathematics at Key Stages 1, 2 and 3. Summary Report*. School Curriculum and Assessment Authority.
- Buchanan, N.K. (1987) Factors Contributing to Mathematical Problem-Solving Performance : An Exploratory Study. *Educational Studies in Mathematics*, 18, 399-415.

Burkhardt, H. (1988) Teaching problem solving. In H.Burkhardt, S.Groves, A.Schoenfeld and K.Stacey, (Eds.) *Problem Solving - A World View: Proceedings of Problem Solving Theme Group, ICME 5*, Shell Centre, University of Nottingham.

Burkhardt, H., Schoenfeld, A., Groves, S. & Stacey, K. (1986) Problem Solving, in Carss, M. (Ed) *Proceedings of the Fifth International Conference on Mathematical Education*. Stuttgart: Birkhäuser.

Burton, L. (1986) Integrating Problem Solving in the Curriculum, in Carss, M. (Ed) *Proceedings of the Fifth International Conference on Mathematical Education*, Stuttgart: Birkhäuser.

Campbell, D.T. and Stanley, J.C. (1963) Experimental and quasi-experimental designs for research on teaching. In N.L.Gage (Ed.) *Handbook of Research on Teaching*. Chicago: Rand McNally.

Campbell, R.J., Evans, L., Neill, S.R. and Packwood, A. (1991). Workloads Achievement and Stress. Two follow-up studies of Teacher Time.in Key Stage 1. Unpublished report commissioned by the Assistant Master and Mistresses Association. Policy Analysis Unit, Department of Education, The University of Warwick. (quoted in Bennett et al. above).

Campione, J.C., Brown, A.L. and Connell, M.L. (1988) Metacognition: On the Importance of Understanding What You Are Doing. In R.I.Charles and E.A.Silver (Eds.), *The Teaching and Assessing of Mathematical Problem Solving*, Reston, Va: Erlbaum/NCTM.

Cardelle-Elawar, M. (1992) Effects of Teaching Metacognitive Skills to Students with Low Mathematics Ability. *Teaching and Teacher Education*. 8, 2, 109-121.

Cardelle-Elawar, M. (1995) Effects of Metacognitive Instruction on Low Achievers in Mathematics Problems. *Teaching and Teacher Education*, 11, 1, 81-95.

Carr, M. and Biddlecomb, B. (1998) Metacognition in Mathematics From a Constuctivist Perspective. In D.J. Hacker, J. Dunlosky and A.C. Graesser, (Eds), *Metacognition in Educational Theory and Practice*. London: Lawrence Erlbaum.

Charles, R.I. and Lester, F.K. (1984a) An evaluation of a process-oriented instructional program in mathematical problem solving in grades 5 and 7. *Journal for Research in Mathematics Education*. 15, 15-34.

Charles, R.I. and Lester, F.K. (1984b) *Teaching Problem Solving*. London: Edward Arnold.

Charles, R.I. and Silver, E.A. (Eds.) (1988) *The teaching and assessing of mathematical problem solving*. Reston, VA: National Council of Teachers of Mathematics.

Chi, M.T.H., Glaser, R. and Farr, M. (1988) *The Nature of Expertise*. Hillsdale, NJ: Erlbaum.

Chinnappan, M. and Lawson, M.J. (1996) The Effects of Training in the Use of Executive Strategies in Geometry Problem Solving. *Learning and Instruction*. 6, 1-17.

Cobb, P., Yackel, E. and Wood, T. (1989) Young Children's Emotional Acts While Engaged in Mathematical Problem Solving. In D.B. McLeod and V.M. Adams, *Affect and Mathematical Problem Solving*. New York: Springer Verlag.

Cobb, P., Yackel, E. and Wood, T. (1992) Interaction and Learning in Mathematics Classroom Situations, *Educational Studies in Mathematics*, 23, 99-122.

Cockcroft, W.H. (1982) *Mathematics Counts: report of the Committee of Enquiry into the Teaching of Mathematics in Schools*. London: HMSO.

Cohen, L. and Manion, L. (1994) *Research Methods in Education*. London: Routledge.

Commission on Standards for School Mathematics (1987) *Curriculum and evaluation standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Committee on Primary Education (COPE) (1983) *Primary Education in the Eighties*. Edinburgh: SCCC.

Cronbach, L. (1963) Course Improvement Through Evaluation. *Teachers' College Record* 64, 672-83.

Daines, J. (1985) Self-evaluation and achievement in a vocationally oriented science practical course. Unpublished PhD dissertation, University of Nottingham.

Dalin, P. (1978) *Limits to Educational Change* New York: Macmillan.

Davidson, J. E. and Sternberg, R. J. (1998) Smart Problem Solving: How Metacognition Helps. In D.J. Hacker, J. Dunlosky and A.C. Graesser, (Eds), *Metacognition in Educational Theory and Practice*. London: Lawrence Erlbaum.

De Corte, E. and Verschaffel, L. (1987). Using retelling data to study young children's word-problem-solving. In J.A.Slododa and D.Rogers, *Cognitive Processes in Mathematics*. Oxford: Clarendon Press.

De Grave, W.S., Boshuizen, H.P.A. and Schmidt, H.G. (1996). Problem Based Learning: Cognitive and Metacognitive Processes During Problem Analysis. *Instructional Science*. 24, 321-341.

DeGuire, L.J. (1987) Awareness of Metacognitive Processes During Mathematical Problem Solving. In J.C. Bergeron, N Herscovics and C.Kieran (Eds.), *Psychology of Mathematics Education. Proceedings of the Eleventh International Conference*. Montreal.

Department for Education and Employment (DfEE), (1999) *The National Numeracy Strategy*. London: Department of Education and Employment.

Descombes, M. and Robins, L. (1980) Self-assessment and essay writing. *Teaching Sociology*. 8, 63-78.

Desforges, C. and Bristow, S. (1995) Academic subjects and pupils' constructive learning processes in the primary school. *British Journal of Educational Psychology*, 65, 465-475.

Devereux, J. (1990) Problem solving with Young Children - A review of the literature. *Early Child Development and Care*. 60, 53-65.

Dewey, J. (1902) *The Child and the Curriculum*, Chicago: University of Chicago Press.

Dewey, J. & Dewey, E. (1915) *Schools of Tomorrow*, New York: Dutton.

Dienes, Z.P. and Jeeves, M.A. (1965) *Thinking in Structures*. London: Hutchison Educational.

Dienes, Z.P. and Jeeves, M.A. (1970) *The Effects of Structural Relations on Transfer*. London: Hutchison Educational.

Dillon, D.R. (1993) The wider social context of innovation in mathematics education. In T.Wood, P.Cobb, E.Yackel and D.Dillon (Eds.) *Rethinking elementary school mathematics: Insights and Issues (Journal for Research in Mathematics Education Monograph Number 6, pp71-96)*. Reston, Va: NCTM.

Eisner, E.W. (1977) On the Uses of Educational Connoisseurship and Criticism for Evaluating Classroom Life *Teachers' College Record* 78, 345-58.

Eisner, E.W. (1978) Humanistic Trends and the Curriculum Field *Curriculum Studies* 10, 197-204.

Eisner, E.W. (1985) *The Art of Educational Evaluation: A Personal View* Philadelphia: The Falmer Press.

Ericsson, K.A. and Simon, H.A. (1980) Verbal Reports As Data, *Psychological Review*, 87, (3), 215-251.

Fitzgerald, T. (1988) Calculators at work and in school. *Mathematics in Schools*. 17, (3), 8-10.

Fitzpatrick, C. (1994) Adolescent Mathematical Problem Solving: The Role of Metacognition, Strategies and Beliefs. Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, La.

Flavell, J.H., Friedrichs, A.G. and Hoyt, J.D. (1970) Developmental changes in memorization processes. *Cognitive Psychology*. 1, 324-340.

Flavell, J.H. (1976) Metacognitive aspects of problem solving. In L. Resnick (Ed). *The Nature of Intelligence..* Hillsdale, NJ: Erlbaum.

Flavell, J.H. and Wellman, H.M. (1977) Metamemory. In R.V.Krail and J.W.Hagen (Eds.), *Perspectives on the development of memory and cognition*. Hillsdale, NJ: Erlbaum.

Flavell, J.H. (1987) Speculations About the Nature and Development of Metacognition. In F.E.Weinert and R.H.Kluwe (Eds.), *Metacognition, Motivation and Understanding*. Hillsdale, NJ: Erlbaum.

Fontana, D. and Fernandes, M. (1994) Improvements in mathematics performance as a consequence of self-assessment in Portuguese primary school pupils. *British Journal of Educational Psychology*, 64, 407-417.

Forman, E. (1989) The role of peer interaction in the social construction of mathematical knowledge. *International Journal of Educational Research*, 13, 55-70.

Fraser, B. J. in collaboration with K. Houghton, (1982) *Annotated bibliography of curriculum evaluation literature*. Jerusalem: Israel Curriculum Centre, Ministry of Education and Culture.

Fraser, B. (1984) Directions in Curriculum Evaluation *Studies in Educational Evaluation*, 10, (2), 125-34.

Gagné, R.M. (1970) *The Conditions of Learning*. New York: Holt, Rhinehart and Winston.

Gardner, H. (1991) *The unschooled mind*. New York, Harper & Row.

- Garofalo, J. and Lester, F.K. (1985) Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education*, 16, (3), 163-176.
- Garofalo, J. Kroll, D.L. and Lester, F.K. (1987) Metacognition and Mathematical Problem Solving: Preliminary Research Findings. In J.C. Bergeron, N.Herscovics and C.Kieran (Eds.), *Psychology of Mathematics Education. Proceedings of the Eleventh International Conference*. Montreal.
- Ginsburg, H.P., Kossan, N.E., Schwartz, R. and Swanson, D. (1983) Protocol methods in research in mathematical thinking. In H. Ginsburg (Ed.), *The Development of Mathematical Thinking*, New York: Academic Press.
- Glass, G.V. (1969) The Growth of Evaluation Methodology *Research Paper No. 27, Boulder, Colorado; Laboratory of Educational Research*; University of Colorado.
- Goldin, G.A. (1982) The Measure of Problem Solving Outcomes. In F.K.Lester, (Ed.), *Mathematical Problem Solving*, Philadelphia, The Franklin Institute Press.
- Goldin, G.A. and McLintock, C.E. (Eds.) (1984) *Task variables in mathematical problem solving*. Hillsdale, N.J: Lawrence Erlbaum.
- Goos, M. and Galbraith, P. (1996) Do it this way! Metacognitive strategies in Collaborative Mathematical Problem Solving. *Educational Studies in Mathematics*. 30, 229-260.
- Green, C. and Gilhooly, K. (1996) Protocol analysis: practical implementation. In J.T.E.Richardson, (Ed) *Handbook of Qualitative Research Methods*. Leicester: BPS Books
- Guba, E.G. & Lincoln, Y.S. (1981) *Effective Evaluation* London: Jossey-Bass.
- Guttentag, M. (1971) Models and Methods in Evaluation Research. *Journal of the Theory of Social Behavior*, 1, 75-95.
- Hacker, D. J. (1998) Definitions and Empirical Foundations. In D.J. Hacker, J. Dunlosky and A.C. Graesser, (Eds), *Metacognition in Educational Theory and Practice*. London: Lawrence Erlbaum.
- Hamdan, M.Z. (1988) Curriculum Evaluation: Toward a Systematic Methodology. *RIE* Nov. 1988.
- Hamilton, D. (1976) *Curriculum Evaluation*. London: Open Books.
- Hamilton, D., Jenkins, D., MacDonald, B., King, C., and Parlett, M. (1977) *Beyond the Numbers Game*. London: Macmillan Educational.
- Hamilton, D. (1990) *Learning about Education: An Unfinished Curriculum* , Milton Keynes: Open University Press.
- Harrap, H. (Ed.) (1937) *The Changing Curriculum*, New York: Appleton-Century.
- Harlen, W. & Malcolm, H.C. (1993) 'Evaluation of the 5-14 Development Programme: An interim report on progress in primary schools' Edinburgh, SCRE.
- Hart, K. (Ed) (1981) *Children's Understanding of Mathematics:11-16*, London: John Murray.
- Hill, J. (1985) Curriculum Evaluation - Practical Approaches to Dealing With the Pitfalls *NASSP Bulletin* 6, 1-6.



Hirabayashi, I. and Shigematsu, K. (1987) Metacognition: The Role of the Inner Teacher. In J.C. Bergeron, N.Herscovics and C.Kieran (Eds.), *Psychology of Mathematics Education. Proceedings of the Eleventh International Conference*. Montreal.

Hissong, C. (1932) *The Activity Movement*, Baltimore: Warwick and York.

Holton, D., Anderson, J. and Thomas, B. (1996) *OPE-N Plan for Teaching Mathematical Problem Solving*. University of Otago.

House, E.R. (Ed) (1973) *School Evaluation: The Politics and the Process* Berkeley: McCutchan.

House, E.R. (1978) Assumptions Underlying Evaluation Models *Educational Researcher*, 7 (3) 4-12.

Jenkins, D. (1976) *Curriculum Evaluation. E203 Units 19, 20 and 21*, Milton Keynes: Open University Press.

Johnson, D.C. and Millett, A. (Eds.) (1996) *Implementing the Mathematics National Curriculum. Policy, Politics and Practice*. London: Paul Chapman.

Kilpatrick, J. (1985) A retrospective account of the past 25 years of research on teaching mathematical problem solving. In E.A. Silver (Ed) *Teaching and learning mathematical problem solving: Multiple research perspectives*. Hillsdale, N.J: Lawrence Erlbaum.

Kilpatrick, J. (1986) Reflection and recursion. In M.Carss (Ed.), *Proceedings of the Fifth International Conference on Mathematical Education*, Stuttgart: Birkhäuser.

Kluwe, R.H. (1987) Executive decisions and regulation of problem solving behaviors (Eds.). In F.E.Weinert and R.H.Kluwe (Eds.), *Metacognition, Motivation and Understanding*. Hillsdale, N.J: Lawrence Erlbaum.

Krejcie, R.V. and Morgan, D.W. (1970) Determining Sample Size for Research Activities, *Educational and Psychological Measurement*, 30, 607-610.

Krulik, S. and Rudnick, J.A. (1988) Helping Teachers Become Teachers of Problem Solving. In H. Burkhardt, S.Groves, A.Schoenfeld and K.Stacey, (Eds.) *Problem Solving - A World View: Proceedings of Problem Solving Theme Group, ICME 5*, Shell Centre, University of Nottingham.

Krutetskii, V. A., (1976) *The Psychology of Mathematical Abilities in School Children*. Chicago: Univ. of Chicago Press.

Lampert, M. (1990) When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal*, 27, 29-63.

Lawson, M.J. and Rice, D.N. (1987) Solving Word Problems: A Detailed Analysis Using Think Aloud Data. In J.C.Bergeron, N.Herscovics and C.Kieran (Eds.), *Psychology of Mathematics Education. Proceedings of the Eleventh International Conference*. Montreal.

Lawton, D. (1982) The Politics of Curriculum Evaluation, in R. McCormick, (Ed), *Calling Education to Account*. London: Heinemann Educational.

Lesh, R. (1985) Conceptual analyses of problem-solving performance. In E.A.Silver (Ed.) *Teaching and Learning Mathematical Problem Solving: multiple research perspectives*. Hillside, NJ: Lawrence Erlbaum Associates.

Lester, F.K. (1982) Building Bridges Between Psychological and Mathematics Education Research on Problem Solving. In F.K.Lester, (Ed.), *Mathematical Problem Solving*, Philadelphia, The Franklin Institute Press.

Lester, F.K. (1983) Trends and issues in mathematical problem solving research. In R.Lesh and M.Landau, (Eds.), *Acquisition of mathematics concepts and processes*. New York. Academic Press.

Lester, F.K. (1985) Methodological considerations in research on mathematical problem solving instruction. In E.A.Silver, (Ed.) *Teaching and learning mathematical problem solving: Multiple research perspectives*. Hillsdale, N.J. Lawrence Erlbaum.

Lester, F.K. (1987) Why is Problem Solving such a problem? Reactions to a Set of Research Papers. In J.C.Bergeron, N.Herscovics and C.Kieran (Eds.), *Psychology of Mathematics Education. Proceedings of the Eleventh International Conference*. Montreal.

Lester, F.K. (1988) Reflections about Mathematical Problem Solving Research, In R.I.Charles and E.A.Silver, (Eds.), *The teaching and Assessing of Mathematical Problem Solving*. Reston, Va. NCTM/Erlbaum.

Lester, F.K. (1994) Musings about Mathematical Problem Solving Research: 1970-1994. *Journal for Research in Mathematics Education*, 25, (6), 660-675.

Lester, F.K. and Garofalo, J. (Eds.) (1982) *Mathematical problem solving: Issues in research*. Philadelphia, Franklin Institute Press.

Lester, F.K., Garofalo, J. and Kroll, D.L. (1989a) Self Confidence, Interest, Beliefs and Metacognition: Key Influences on Problem Solving Behavior. In D.B.McLeod and V.M. Adams, (Eds.), *Affect and Mathematical Problem Solving*. New York, Springer Verlag.

Lester, F.K., Garofalo, J. and Kroll, D.L. (1989b) The Role of Metacognition in Mathematical Problem Solving: A Study of Two Grade Seven Classes. Final Report. Indiana University, Bloomington. Mathematics Education Development Center.

Lewy, A. (1973) The practice of curriculum evaluation. *Curriculum Theory Network*, 11 (spring), 6-33.

Lindvall, C.M. & Cox, R.C. (1970) The IPI Evaluation Program. *AERA Monograph Series on Curriculum Evaluation* No 5 Chicago: Rand McNally.

Logan, J.L. (1995) 5-14 Mathematics Programme Evaluation. Interim Report of the Pilot Study. Unpublished Manuscript. Northern College of Education.

Logan, J.L. (1996) Have we solved the problem of problem solving *Scottish Mathematical Council, Journal*, 25 ,26-30.

Lublin, J. (1980) Student self-assessment. A case study. *Assessment in Higher Education*, 5, 264-272.

McCormick, R. (Ed) (1982) *Calling Education to Account*. London: Heinemann Educational.

MacDonald, B. (1971) Evaluation of the Humanities Curriculum Project: A holistic approach. *Theory into Practice*, 10 (3) 163-167.

MacDonald, B. (1973) Briefing Decision Makers, in E.R. House, (Ed) *School Evaluation: The Politics and the Process*, Berkeley: McCutchan.

- MacDonald, B. (1976) Evaluation and the Control of Education, in D. Tawney, (Ed) *Curriculum Evaluation Today: Trends and Implications*, London: Macmillan.
- McLeod, D.B. (1989) The Role of Affect in Mathematical Problem Solving. In D.B.McLeod and V.M.Adams, *Affect and Mathematical Problem Solving*. New York: Springer Verlag.
- McLeod, D.B. (1994) Research on Affect and Mathematics Learning in the JRME: 1970 To The Present. *Journal for Research In Mathematics Education*, 25, 6, 637-647.
- McNamara, D. (1990) The National Curriculum: An Agenda for Research *British Educational Research Journal*, 16, (3), 225-235.
- Mandler, G. (1984) *Mind and body: Psychology of emotion and stress*. New York: Norton.
- Mandler, G. (1989) Affect and learning causes and consequences of emotional interactions. In D.B.McLeod and V.M.Adams, *Affect and Mathematical Problem Solving*. New York: Springer Verlag.
- Marshall, S.P. (1988) Assessing Problem Solving: A Short-Term Remedy and a Long-Term Solution. In R.I.Charles and E.A.Silver (Eds.), *The Teaching and Assessing of Mathematical Problem Solving*. Reston, Va: Erlbaum/NCTM.
- Mason, J., Burton, L. and Stacey, K. (1985) *Thinking Mathematically*, (Revised ed.) Wokingham: Addison Wesley.
- Matsushita, K. (1994) Acquiring Mathematical Knowledge through Semantic and Pragmatic Problem Solving. *Human Development*. 37, 220-232.
- Mayer, R.E. (1985) Implications of Cognitive Psychology for Instruction in Mathematical Problem Solving. In E.A.Silver (Ed.), *Teaching and Learning Mathematical Problem Solving: Multiple Research Perspectives*. Hillsdale, NJ: Erlbaum.
- Mayer, R.E. (1987) *Educational Psychology*. Boston: Little, Brown and Co.
- Meyer, M.R. and Fennema, E. (1988) Girls, boys and mathematics. In T.R. Post (Ed.) *Teaching mathematics in grades K-8: Research-based methods*. Boston: Allyn and Bacon.
- Millett, A. and Askew, M. (1994) Teachers' Perceptions of Using and Applying Mathematics. *Mathematics Teaching*. 148, 3-7.
- Millett, A. and Johnson, D.C. (1996) Solving Teachers' Problems? The Role of the Commercial Scheme. in Johnson et al (Eds) *Implementing the National Curriculum*. London: Paul Chapman.
- National Council of Supervisors of Mathematics. (1977) Position paper on basic skills. *Arithmetic Teacher*. 25, 1, 19-22.
- National Council of Teachers of Mathematics, (1980) *An agenda for action: recommendations for school mathematics of the 1980s*. Reston, Va: NCTM.
- National Curriculum Council (1989) *Mathematics. Non-Statutory Guidance*, NCC, York.
- National Curriculum Council (1990) *Report on Monitoring the Implementation of the National Curriculum Core Subjects 1989-1990*, York: NCC.
- National Education Association (NEA)(1960) Small-Sample Techniques. *The NEA Research Bulletin*, 38, 99.

- National Numeracy Project (1997) Reading, National Centre for Literacy and Numeracy.
- Nesher, P and Hershkowitz, S. (1994) The Role of Schemes in Two-step Problems: Analysis and Research Findings. *Educational Studies in Mathematics*. 26, 1-23.
- Newell, A. and Simon, H.A. (1972) *Human Problem Solving*. Englewood Cliffs, N.J: Prentice-Hall.
- Nicholls, J. and Miller, A. (1983) The differentiation of the concepts of difficulty and ability. *Child Development*. 54, 951-959.
- Nisbet, J.D. (1974) Educational Research - The State of the Art - *Paper presented to the inaugural meeting of BERA*, Manchester; 1974.
- Nisbet, J. and Shucksmith, J. (1986) *Learning Strategies*. London: Routledge.
- Nisbet, J. (1999) Personal communication.
- Nisbett, R.E. and Wilson, T.D. (1977) Telling More Than We Can Know: Verbal Reports on Mental Processes, *Psychological Review*, 84, 3, 231-259.
- Nixon, J. (1989) Curriculum Evaluation: Towards the Vanishing Point *Westminster Studies in Education*, 12, 91-98.
- Nixon, J. (1992) *Evaluating the Whole Curriculum* Milton Keynes: Open University Press.
- Noddings, N. (1985) Small Groups as a Setting for Research on Mathematical Problem Solving. In E.A.Silver (Ed.), *Teaching and Learning Mathematical Problem Solving: Multiple Research Perspectives*. Hillsdale, NJ: Erlbaum.
- Office for Standards in Education (Ofsted) (1993) *The Teaching and Learning of Numbers in Primary School*. LONDON: HMSO.
- Owings, R., Petersen, G., Bransford, J., Morris, D. and Stein, B. (1980) Spontaneous monitoring and regulation of learning: a comparison of successful and less successful fifth graders. *Journal of Educational Psychology*, 72, 250-256.
- Parlett, M. & Hamilton, D. (1976) Evaluation as Illumination, in D. Tawney, (Ed) *Curriculum Evaluation Today: Trends and Implications*, London: MacMillan.
- Parsons, C. (1976) The New Evaluation: A Cautionary Note *Journal of Curriculum Studies* , 8 (2) 125-38.
- Peterson, P.L., Carpenter, T. and Fennema, E. (1989) Teachers' Knowledge of Students' Knowledge in Mathematics Problem Solving: Correlational and Case Analyses. *Journal of Educational Psychology*. 81, 4, 558-569.
- Peterson, P.L., Fennema, E. and Carpenter, T. (1988) Using Knowledge of How Students Think About Mathematics. *Educational Leadership*. Dec 1988. 42-46.
- Peterson, P.L. and Swing, S.R. (1982) Beyond Time on Task: Students' Reports of Their Thought Processes during Classroom Instruction. *The Elementary School Journal*. 82, 5. 480-491.
- Peterson, P.L., Swing, S.R., Braverman, M.T. and Buss, R. (1982) Students' Aptitudes and Their Reports of Cognitive Processes During Direct Instruction. *Journal of Educational Psychology*, 74, 4, 535-547.

- Piaget, J. (1969) *Judgment and Reasoning in the Child*. London: Routledge and Kegan Paul. (Original work published 1928).
- Piaget, J. (1976) *The Grasp of Consciousness*. Cambridge, MA: Harvard Univ. Press.
- Polya, G. (1945) *How to Solve It*. Princeton, N.J: Princeton Univ. Press.
- Popham, W.J. (1975) *Educational Evaluation*. Englewood Cliffs N.J: Prentice Hall.
- Popham, W.J. (1977) Must All Objectives be Behavioral?, in D. Hamilton, et al.(Eds.) *Beyond the Numbers Game*. London: Macmillan.
- Pramling, I. (1988) Developing Children's Thinking About Their Own Learning. *British Journal of Educational Psychology*, 58, 266-278.
- Pressley, M. (1994) Embracing the complexity of individual differences in cognition: Studying good information processing and how it might develop. *Learning and Individual Differences*, 6, 259-284.
- Quinto, A.L. and Weener, P.D. (1983) Assessing Metacognitive Skills in Problem Solving. Paper presented at the Annual Meeting of the American Educational Research Association, Montreal.
- Randhawa, B.S. (1994) Theory, Research and Assessment of Mathematical Problem Solving. *The Alberta Journal of Educational Research*. XL, 2, 213-231.
- Reichhardt, C.S. & Cook, T.D. (1979) Beyond Qualitative versus Quantitative Methods, in C.S. Reichhardt, & T.D. Cook, (Eds) *Qualitative and Quantitative Methods in Evaluation Research*, Beverly Hills: Sage.
- Resnick, L.B. (1988) Treating Mathematics as an Ill-Structured Discipline. In R.I.Charles and E.A.Silver, *The Teaching and Assessing of Mathematical Problem Solving*, Reston, Va: NCTM/Erlbaum.
- Roberts, M.J. (1991) Individual differences and strategy selection in problem solving. *Unpublished doctoral dissertation*, University of Nottingham.
- Roberts, M.J. and Erdos, G. (1993) Strategy Selection and Metacognition. *Educational Psychology*, 13, 3 and 4, 259-266.
- Ruberu, J. (1983) Curriculum Evaluation and its Relevance to Mathematics Education *Journal of Science and Maths Education in S.E. Asia*, 6 (2), 28-35.
- Rugg, H. and Shumaker, A. (1928) *The Child-centred School*, New York: World Book Co.
- SCAA (1993) *Evaluation of the Implementation of National Curriculum Mathematics at Key Stages 1, 2 and 3. Volume 1: Report*. London: SCAA.
- Schoenfeld, A.H. (1979) Explicitly Heuristic Training as a Variable in Problem Solving Performance. *Journal for Research in Mathematics Education*. 10, 173-187.
- Schoenfeld, A.H. (1981) *Episodes and executive decisions in mathematical problem solving*. Paper presented at the meeting of the American Educational Research association, Los Angeles.
- Schoenfeld, A.H. (1982) Some Thoughts on Problem Solving Research and Mathematics Education. In F.K. Lester (Ed.). *Mathematical Problem Solving*. Philadelphia: The Franklin Institute Press.

Schoenfeld, A.H. (1983) Episodes and executive decisions in mathematical problem solving. In R.Lesh & M.Landau (Eds.), *Acquisition of mathematics concepts and processes*. New York: Academic Press.

Schoenfeld, A.H.(1984) Beyond the purely cognitive: Belief systems, social cognitions and metacognitions as driving forces in intellectual performance. *Cognitive Science*, 7, 329-363.

Schoenfeld, A.H. (1985a) *Mathematical problem solving*. Orlando, FL: Academic Press.

Schoenfeld, A.H. (1985b) Artificial Intelligence and Mathematics Education: A Discussion of Rissland's Paper. In E.A.Silver (Ed.) *Teaching and Learning Mathematical Problem Solving: Multiple research perspectives*. Hillsdale, N.J: Lawrence Erlbaum.

Schoenfeld, A.H. (Ed.) (1987a) *Cognitive science and mathematics education*. Hillsdale, N.J: Lawrence Erlbaum.

Schoenfeld, A.H. (1987b) What's all the fuss about metacognition? In A.H.Schoenfeld (Ed.), *Cognitive science and mathematics education*, Hillsdale, N.J: Lawrence Erlbaum.

Schoenfeld, A.H. (1988) Research on Problem Solving, in Burkhardt, H. et al. (Eds) *Problem Solving - A World View: Proceedings of Problem Solving Theme Group, ICME 5*, Shell Centre, Univ. of Nottingham.

Schoenfeld, A.H. (1992) Learning to Think Mathematically: Problem Solving, Metacognition and Sense Making in Mathematics. In D.A.Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning*. New York: Macmillan.

School Curriculum and Assessment Authority (1993) *Evaluation of the Implementation of National Curriculum Mathematics at Key Stages 1, 2 and 3. Volume 1: Report* SCAA London.

Schultz, K. (1987) Theoretical Perspectives on Inquiry into Problem Solving, Knowledge Acquisition, Metacognition and Teaching. In J.C.Bergeron, N.Herscovics and C.Kieran (Eds.), *Psychology of Mathematics Education. Proceedings of the Eleventh International Conference*. Montreal.

Scottish Consultative Council on the Curriculum (SCCC) (1993a) *Mathematics 5-14 Exemplification*. Dundee. SCCC.

Scottish Consultative Council on the Curriculum (SCCC) (1993b) *5-14 Catalogue: Mathematics*. Dundee. SCCC.

Scottish Education Department (SED)(1987) *Curriculum and Assessment in Scotland. A policy for the 90s*. Edinburgh. The Scottish Education Department.

Scottish Education Department (SED)(1990) *Curriculum and Assessment in Scotland. A policy for the 90s: Working Paper No 3*. Edinburgh. The Scottish Education Department.

Scottish Examination Board (SEB) (1984) *Standard Grade. Arrangements in Mathematics At Foundation, General and Credit Levels*. Dalkeith. The Scottish Examination Board.

Scottish Office Education Department (SOED)(1991) *Curriculum and Assessment in Scotland: National Guidelines: Mathematics 5-14*. Edinburgh: The Scottish Office Education Department.

Scottish Office Education Department (SOED)(1993) *The Structure and Balance of the Curriculum 5-14*. Edinburgh. The Scottish Office Education Department.

Scottish Office Education Department (SOED)(1994) *5-14: A Practical Guide for teachers in primary and secondary schools*. Edinburgh. The Scottish Office Education Department.

Scottish Office Education Department (SOED) (1995) *Scottish Education Statistics and Fact Card 1995*. Edinburgh. The Scottish Office Education Department.

Scottish Office Education and Industry Department (SOEID)(1996a) *Assessment of Achievement Programme: Mathematics 1994*. Edinburgh: The Scottish Office Education and Industry Department.

Scottish Office Education and Industry Department (SOEID)(1996b) *Achievements of Primary 4 and Primary 5 Pupils in Mathematics and Science: Third International Mathematics and Science Study (TIMSS)*. Edinburgh: The Scottish Office Education and Industry Department.

Scottish Office Education and Industry Department (SOEID)(1997) *Improving Mathematics Education 5-14. A Report By HM Inspectors of Schools*. Edinburgh: The Scottish Office Education and Industry Department.

Scriven, M. (1967) *The Methodology of Evaluation. AERA Monograph Series in Curriculum Evaluation, No 1*, Chicago: Rand McNally.

Scriven, M. (1972a) *Prose and Cons About Goal Free Evaluation Education Comment* 3, 1-4.

Scriven, M. (1972b) *Objectivity and Subjectivity in Educational Research*, in L.G. Thomas, (Ed), *Philosophical Redirection of Educational Research*, Chicago: NSSE/University of Chicago Press.

Scriven, M. (1973) *Goal Free Evaluation*, in E.R. House, (Ed), *School Evaluation: the Politics and the Process*, Berkeley: McCutchan.

Scriven, M. (1974) *Evaluation Perspectives and Procedures*, in W.J. Popham, (Ed), *Evaluation in Education: Current Applications*, Berkeley: McCutchan.

Sewell, B. (1981) *The Use of Mathematics by Adults in Daily Life*, Leicester: ACACE.

Shuard, H. (1986) *Towards 2000* Lecture given to 'National In-Service Course on 10-14 Mathematics'. Dundee College of Education.

Shuard, H. Walsh, A., Goodwin, J. and Worcester, V. (1991) *Calculators, Children and Mathematics*. London: Simon & Schuster.

Silver, E.A. (1982a) *Thinking about problem solving: Toward an understanding of metacognitive aspects of mathematical problem solving*. Paper prepared for the Conference on Thinking, Fiji.

Silver, E.A. (1982b) *Knowledge Organisation and Mathematical Problem Solving*. In F.K.Lester (Ed.), *Mathematical Problem Solving*. Philadelphia: The Franklin Institute Press.

Silver, E.A. and Thompson, A.G. (1984) *Research Perspectives on Problem Solving in Elementary School Mathematics. The Elementary School Journal*. 84, 5, 529-545.

Silver, E.A. (Ed.) (1985) *Teaching and learning mathematical problem solving: Multiple research perspectives*. Hillsdale, N.J: Lawrence Erlbaum.

Silver, E.A. (1987) Perusing the Problem solving Panorama: Comments on Six Papers on Mathematical Problem Solving. In J.C.Bergeron, N.Herscovics and C.Kieran (Eds.), *Psychology of Mathematics Education. Proceedings of the Eleventh International Conference*. Montreal.

Silver, E.A. (1988) Problem Solving, Proof and Process Aspects of Mathematics. In H.Burkhardt, S.Groves, A.Schoenfeld and K.Stacey, (Eds.), *Problem Solving - A World View: Proceedings of Problem Solving Theme Group, ICME 5*, Shell Centre, University of Nottingham.

Silver, E.A. and Kilpatrick, J. (1988) Testing Mathematical Problem Solving. In R.I.Charles and E.A.Silver (Eds.), *The Teaching and Assessing of Mathematical Problem Solving*. Reston, Va: Erlbaum/NCTM.

Skemp, R. (1979) *Intelligence, learning and action*. New York: Wiley.

Smith, E.R. and Miller, F.S. (1978) Limits on Perception of Cognitive Processes: A Reply to Nisbett and Wilson, *Psychological Review*, 85, 4, 355-362.

Socket, H. (1982) Accountability: Purpose and Meaning. In R. McCormick,(Ed), *Calling Education to Account*, London: Heinemann Educational.

Stacey, K. and Groves, S. (1984) Problem Solving: People and Projects in Australia. In P. Costello, S. Ferguson, K. Slinn, M. Stephens, D. Trembath and D Williams (Eds.) *Facets of Australian Mathematics Education*. Blackburn. Australian Association of Mathematics Teachers.

Stacey, K. and Groves, S. (1988) Curriculum Development in Problem Solving. In H. Burkhardt, S. Groves, A. Schoenfeld and K. Stacey, (Eds.) *Problem Solving - A World View: Proceedings of Problem Solving Theme Group, ICME 5*, Shell Centre, University of Nottingham.

Stake, R.E. (1967a) The Countenance of Educational Evaluation *Teachers' College Record* 68, 523-40.

Stake, R.E. (1967b) Toward a Technology for the Evaluation of Educational Programs, in R. Tyler, et al. (Eds) *Perspectives of Curriculum Evaluation*, *AERA Monograph Series on Curriculum Evaluation No 1*, Chicago: Rand McNally.

Stake, R.E. (1972) Analysis and Portrayal *Paper presented to AERA Annual Meeting 1972*.

Stake, R.E. (1975) *Evaluating the Arts in Education: A Responsive Approach* , Columbus: Merrill.

Stanic, G. and Kilpatrick, J. (1988) Historical perspectives on problem solving in the mathematics curriculum. In R.Charles and E.A.Silver (Eds.) *The Teaching and Assessing of Mathematical Problem Solving*..Reston, Va: NCTM.

Steadman, S. (1976) Techniques of Evaluation, in D. Tawney, (Ed.) *Curriculum Evaluation Today: Trends and Implications*, London: Macmillan.

Stenhouse, L. (1979) The Problem of Standards in Illuminative Research: The 1979 SERA lecture, *Scottish Educational Review* , 11 (1), 5-11.

Stone, C.A. (1985) Vygotsky's developmental model and the concept of proleptic instruction: Some implications for theory and research in the field of learning disabilities. *Research Communications in Psychology, Psychiatry and Behavior*, 10, 1&2, 129-152.



Stufflebeam, D. et al. (1971) Educational Evaluation and Decision Making. In *Phi Delta Kappa National Study Committee on Evaluation* 1971.

Stufflebeam, D. (1972) Should or Can Evaluation be Goal Free? *Evaluation Comment* , 3 (4).

Sutton, R.E. (1991) Equity and Computers in the School: A decade of Research. *Review of Educational Research*, 61, (4), 475-503.

Sweller, J. (1990) On the limited evidence for the effectiveness of teaching general problem solving strategies. *Journal for Research in Mathematics Education*, 21, 411-415.

Tellep, M.M. (1989) Curriculum Evaluation: An Historical Approach *Research in Education* , Sept. 1989.

Thompson, A.G. (1985) Teachers' Conceptions of Mathematics and the teaching of Problem Solving. In E.A.Silver (Ed.) *Teaching and Learning Mathematical Problem Solving: Multiple Research Perspectives*. Hillsdale, NJ. Erlbaum.

Thompson, A.G. and Thompson, P.W. (1989). Affect and Problem Solving in an Elementary School Mathematics Classroom. In D.B.McLeod and V.M.Adams, *Affect and Mathematical Problem Solving*. New York. Springer Verlag.

Thompson, I. (1993) What Kate Did Next, *Times Educational Supplement*, March 12 1993.

TIMSS (1997) *Third International Mathematics and Science Study. Achievements of Primary 4 and Primary 5 Pupils in Mathematics and Science*.  
Edinburgh: S.O.E.I.D.

Tyler, R. (1949) *Basic Principles of Curriculum and Instruction*. Chicago: University of Chicago Press.

Tyler, R., Gagne, R. & Scriven, M. (1967) Perspectives of Curriculum Evaluation *AERA Monograph Series on Curriculum Evaluation No 1*, Chicago: Rand McNally.

Usnick, V.E. and Brown, S. (1992) Assessing Elementary Students' Thinking Through Interpersonal Process Recall. *Focus on Learning Problems in Mathematics*, Fall Edition, 14, 4, 56-62.

VanLehn, K. (1991) Rule acquisition events in the discovery of problem solving strategies. *Cognitive Science*, 15, 1-47.

Vygotsky, L.S. (1978) *Mind in Society: The development of higher psychological processes*. Cambridge, MA; Harvard University Press.

Walker, D. and Schaffarzick, J. (1974) Comparing Curricula. *Review of Educational Research* 44 (1) 83-111.

Watson, G. (Ed) (1967) Change in School Systems. *Cooperative Project for Educational Development*; National Education Association.

Weinert, F.E. (1987) Introduction and Overview: Metacognition and Motivation as Determinants of Effective Learning and Understanding. In F.E.Weinert and R.H.Kluwe, (Eds.), *Metacognition, Motivation and Understanding*, Hillsdale, NJ: Erlbaum.

Wertsch, J. (1979) *The social origins of metacognition*. Paper presented at the meeting of the Society for Research in Child Development, San Francisco.

West, R.W. (1975) The Summative Evaluation of Curriculum Innovations. *Occasional Paper 1*, Education Area, University of Sussex.

Whitebread, D. (1996) The development of children's strategies on an inductive reasoning task. *British Journal of Educational Psychology*. 66, 1-21.

Wideen, M. (1986) The Role of the Evaluator. *Cambridge Journal of Education* 16 (2) 126-30.

Williams, D., Wilson, K., Richardson, A., Tuson, J. and Coles, L. (1998) *Teachers' ICT Skills and Knowledge Needs*. Aberdeen: The Robert Gordon University.

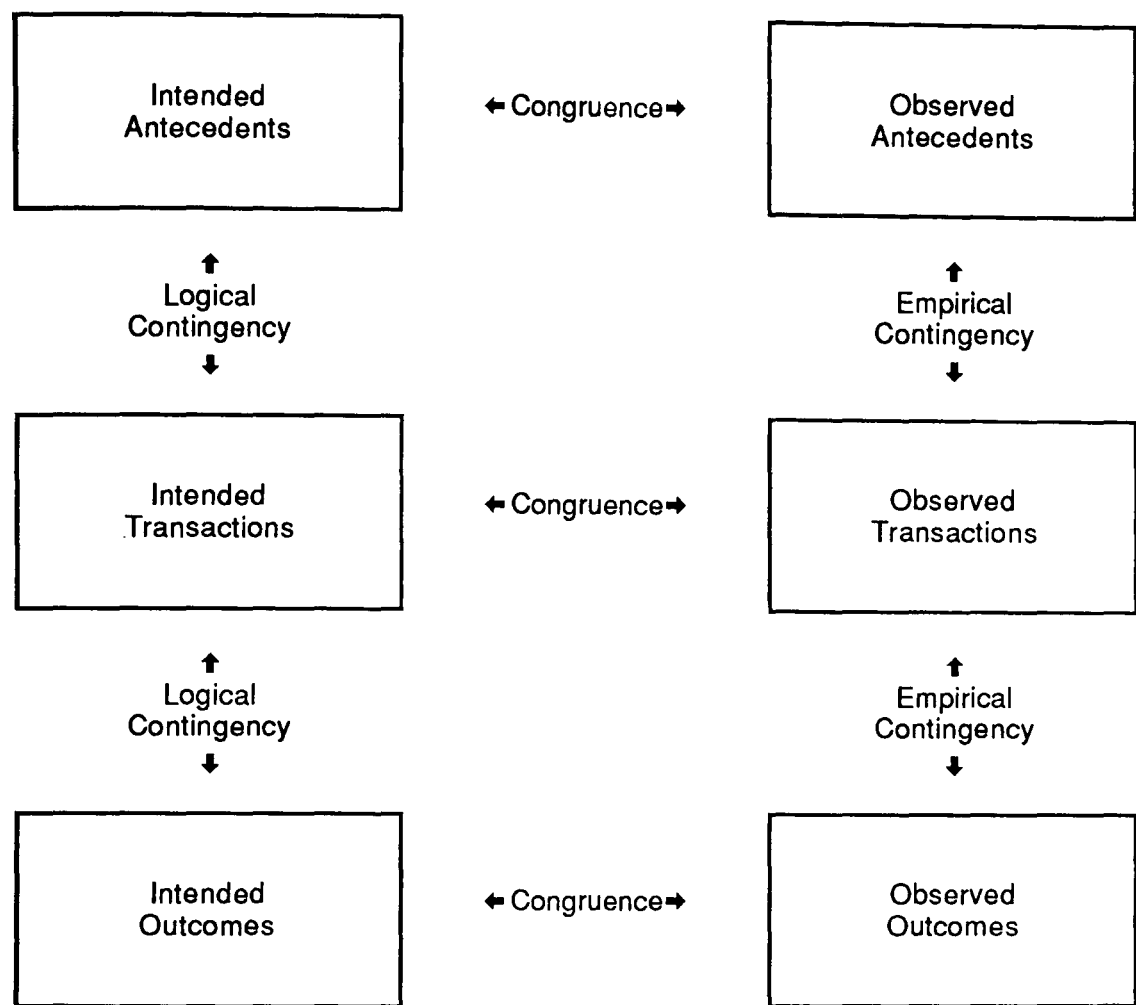
Wiseman, S. & Pidgeon, D. (1970) Curriculum Evaluation. Slough: NFER.

Wragg, E.C., Bennett, S.N. and Carré, C.G. (1989) Primary Teachers and the National Curriculum. *Research Papers in Education*, 4 (3), 17-37.

Wright, B.D. and Stone, M.H. (1979) *Best Test Design. Rasch Measurement*. Chicago: Mesa Press.

Zaltman, G., Florio, D. & Sikorski, L. (1977) *Dynamic Educational Change*. New York: Free Press.

Stake's (1967) Description Matrix



The 'Intended' column of Stake's descriptive matrix was to be completed with objectives and for each objective, data had to be collected which showed the extent to which the objective had been achieved. The bottom row of this matrix corresponds to Tyler's model, but Stake's went further in that it included information to show whether antecedent conditions and transaction processes had been fulfilled as specified.

Stake's description/judgement matrix

Intents	Observations

Description matrix

Antecedents

Transactions

Outcomes

Standards	Judgements

Judgement matrix

Stufflebeam's CIPP model

<u>Intended Ends</u>	<u>Actual Ends</u>
Planning decisions to identify objectives	Recycling decisions to judge and react to attainments
<u>Context</u> evaluation	<u>Product</u> evaluation
<u>Intended Means</u>	<u>Actual Means</u>
Structuring decisions to design procedures	Implementing decisions to use and refine procedures
<u>Input</u> evaluation	<u>Process</u> evaluation

Context (C) evaluation assesses needs and conditions to provide a rationale for determining objectives. Input (I) evaluation assesses alternative means of achieving the specified ends. Process (P) evaluation monitors the implementation decisions to attempt to keep the processes as close as possible to the intended means. Product (P) evaluation compares, in a Tylerian way, actual with intended ends.

### AN EVALUATION OF THE EFFECTS OF THE IMPLEMENTATION OF THE 5-14 NATIONAL GUIDELINES IN MATHEMATICS

#### Questionnaire for Primary School Teachers

Dear Colleague

This questionnaire is part of an evaluation of the 5-14 National Guidelines in mathematics being conducted by Lindsay Logan at Northern College in Dundee and which is being supported by a grant from the SOEID. The purpose of this questionnaire is to find out the views of practising classroom teachers about the implementation of 5-14 Mathematics and any changes it has brought about in their teaching.

Please note that it is the Mathematics 5-14 Guidelines which are being evaluated, not the teachers who are implementing it, so please take this opportunity of expressing your views fully. You may find some of the questions difficult to complete, but please do the best you can, as we wish to get as complete a picture as possible of teachers' views. It is anticipated that many teachers will have had some difficulties with some aspects of the Guidelines, so please do not hesitate to convey your feelings as explicitly as possible, so that those parts of the Guidelines which have caused difficulties may be identified. All the information you give will remain strictly confidential and will be read only by the research director.

Please return the completed questionnaire, sealed in the stamped addressed envelope provided. Thank you for helping with this evaluation.

Your cooperation is much appreciated.

November 1996

NOTE: THROUGHOUT THIS QUESTIONNAIRE THE WORD 'GUIDELINES' REFERS ONLY TO THE 'NATIONAL GUIDELINES - MATHEMATICS 5-14'

**IMPLEMENTATION OF THE GUIDELINES**

1. How would you describe the stage of implementation you have reached.  
(Please tick the appropriate box)

- (a) An early stage of implementation

☐
- (b) Implementation in progress but not complete

☐
- (c) Implementation nearly complete

☐
- (d) Implementation complete

☐

**YOUR VIEWS OF THE GUIDELINES**

2. What are your views about the guidelines  
(Please tick the appropriate box)

- (a) I like them very much

☐
- (b) I like them a little

☐
- (c) I have no firm views on them

☐
- (d) I dislike them a little

☐
- (e) I dislike them very much

☐

3. To help to provide some ideas of the reasons for your answers to question 2, please tick the box which most nearly matches your views.

	strongly agree	agree	disagree	strongly disagree
(a) The guidelines make sensible recommendations	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(b) They are easy to read	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(c) They attempt to relate maths to real life	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(d) They encourage children to use and apply mathematics to a greater extent than before	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(e) The targets are realistic	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

other reasons, (please specify)

EFFECT OF THE GUIDELINES ON YOUR CLASSROOM PRACTICE

4. Please indicate the effects of the implementation of the Guidelines on each of the following aspects of your work in mathematics.

	no effect at all	minor changes	major changes
(a) The mathematical content you teach	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(b) The ways in which you teach maths	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(c)The ways in which you plan your mathematics teaching	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(d) The ways in which you assess pupils' work in mathematics	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(e) The ways in which you record pupils' mathematics attainments	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

5. In each aspect of your work where changes have taken place, please specify briefly, what these changes were.

(a) In mathematical content:

minor changes

major changes

(b) In the ways in which you teach mathematics:

minor changes

major changess

(c) In the ways in which you plan mathematics:

minor changes

major changes

(d) In the ways in which you assess pupils' work in mathematics:

minor changes

major changes

(e) In the ways in which you record pupils' mathematical attainments:

minor changes

major changes



**SUPPORT AVAILABLE TO HELP YOU WITH THE IMPLEMENTATION OF THE GUIDELINES**

6. Please indicate which of the following forms of support you have had and how useful each has been. (Please tick the 'Yes' box and one other box for each form of support you have had).

	Yes	very useful	quite useful	of limited use	not useful
(a) In-service courses delivered by Regional staff	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(b) In-service courses delivered by College of Education Staff	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(c) Advice from Regional advisers or other Regional personnel who have come to your school	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(d) Informal conversations with colleagues about the guidelines	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(e) Organised meetings with colleagues from your school	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(f) Extra time to yourself for reading, planning and thinking e.g.P.A.T. time	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(g) Advice from your school's promoted staff	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(h) The staff development pack "Mathematics (5-14) Exemplification"	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(i) "5-14 Catalogue : Mathematics" (SCCC 1993b)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(j) "5-14: A Practical Guide" (SOED 1994)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(k) Printed material produced by your Region e.g. 5-14 Regional Guidelines	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(l) Appropriate commercially published material	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Other support you have had (please specify)

7. What further support or resource do you most need to help you implement the guidelines?

8. What factor(s) have caused you the greatest difficulties in implementing the guidelines?

**YOUR VIEWS ON THE PROBLEM-SOLVING AND ENQUIRY OUTCOME**

9. Please tick the box which best matches your level of agreement with each of the following statements about the problem-solving and enquiry (P.S.E.) outcome.

	strongly agree	agree	disagree	strongly disagree
(a) The P.S.E. outcome was one of the most difficult aspects of the guidelines for me	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(b) I feel that I cope well with matching P.S.E. tasks to children	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(c) I would appreciate more guidance about progression in P.S.E.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(d) My own background in maths makes me feel insecure about teaching P.S.E.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(e) I am satisfied with what I am doing with my class in P.S.E.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(f) I would have liked examples of P.S.E. with levels attached	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

10. The guidelines refer (on page 48) to three aspects of the P.S.E. outcome They are:

- 1. Adopting an investigative approach to learning concepts, facts and techniques
- 2. Working on tasks designed specifically to highlight the merits of certain approaches to mathematical thinking
- 3. Using their mathematics in an enquiry which could be part of a cross-curricular study

The following questions refer to these three aspects of problem-solving and your views on them. Please tick the box which most nearly matches your own view.

	strongly agree	agree	disagree	strongly disagree
(a) I am clear about the three aspects of P.S.E.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(b) My teaching programme in P.S.E. includes all three aspects	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(c) The examples in the guidelines (p.48) which illustrate these aspects are helpful	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(d) Which of the three aspects do you feel most comfortable with in your teaching? (Please tick the appropriate box(es))				
Aspect 1				<input type="checkbox"/>
Aspect 2				<input type="checkbox"/>
Aspect 3				<input type="checkbox"/>

11. At what stage are you, with respect to each of the following P.S.E. activities?  
(tick one box to show what stage you are at and indicate with a tick if you would appreciate advice)

	not yet started	started but not far on	reasonably well on	would appreciate some advice
(a) Developing a structured programme in P.S.E. for the year	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(b) Matching P.S.E. activities to 5-14 levels	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(c) Differentiating the P.S.E. activities used in my class	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(d) Developing a procedure for assessing my pupils' P.S.E. work	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(e) Producing a recording and reporting format for each pupil P.S.E. attainments	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

12. The Guidelines, (p12), describe 3 steps in the problem solving process as Starting, Doing and Reporting. This question concerns what your pupils do in the **Reporting** stage. In each case please tick the box which best describes what your pupils do.

They report on each problem they have solved by:

	always	often	occasionally	never
(a) keeping a record of their solutions in their jotters or folders	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(b) explaining to their group (or class) how they did it	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(c) explaining to the teacher how they did it	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(d) referring orally to the strategy (or strategies) used	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(e) writing down the strategy (or strategies) used	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(f) displaying their solutions	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

13. How much time, on average, does each pupil in your class spend on P.S.E. in a week?

less than 30 mins	<input type="checkbox"/>
between 30 mins and 1 hour	<input type="checkbox"/>
more than 1 hour (please specify)	<input type="checkbox"/>

14. How do you feel about the length of time your pupils spend on P.S.E.?

too little	<input type="checkbox"/>
about right	<input type="checkbox"/>
too much	<input type="checkbox"/>

Could you give reasons for your response

15. Which P.S.E. skills, strategies or attitudes do you hope will have been acquired by the end of this academic year by

- (a) your highest attaining pupils?
- (b) your lowest attaining pupils?

YOUR VIEWS ON THE SPECIFIC ISSUE OF CALCULATING

16. The guidelines refer to three methods of calculating - mental, written and using a calculator. This question seeks to determine the extent to which the relative amounts of time spent on each method has changed as a result of the implementation of the guidelines. Think about the time your class spent doing each method of calculating before the guidelines appeared and that spent after their implementation. Try to estimate the percentage of the total time spent on calculating which was (is) allocated to each method.

(Note, In estimating the proportion of time spent on mental calculation, please do not include time spent doing mental recall of number facts).

pre 5-14 practice

- (a) written

%

 ;
- calculator

%

 ;
- mental

%

practice after 5-14 implementation

- (b) written

%

 ;
- calculator

%

 ;
- mental

%

17. How do you feel about the proportions in 15(b) above (tick appropriate box)

- (a) about right
- (b) in need of review

If you ticked (b), please specify how you think the proportions should be changed.

18. Think of the numerical calculations you perform in real life, but do not count your professional activities as a teacher, and estimate the percentage of your total calculations done in each way.

- written

%

 ;
- calculator

%

 ;
- mental

%

19. Do you ever encourage your pupils to devise and use **personal** methods of mental calculation?

YES

☐

NO

☐

If NO, please go to question 20

If YES, please indicate for which group (or groups) of pupils you do this.

(a) all pupils

☐

(b) all except the lowest attainers

☐

(c) all except the highest attainers

☐

(d) only the lowest attainers

☐

(e) only the highest attainers

☐

(f) other (please specify)

20. Think about the **mental** method **you** would use to perform each of the following calculations.

(a)  $120 - 87$

(b)  $99 \times 6$

(c)  $78 + 13$

For each of the mental methods you used, tick the appropriate box to show how you learned it.

I was taught it at school

(a)

☐

(b)

☐

(c)

☐

I worked it out for myself

☐☐☐

don't know

☐☐☐

21. Do you ever encourage your pupils to use personal, non-standard methods of **written** calculation?

YES

☐

NO

☐

If NO, could you say why not?

If YES, please indicate for which group (or groups) of pupils you do this?

(a) all pupils

☐

(b) all except the lowest attainers

☐

(c) all except the highest attainers

☐

(d) only the lowest attainers

☐

(e) only the highest attainers

☐

(f) other (please specify)

22. This question seeks to identify changes in your classroom use of calculators as a result of the implementation of the guidelines (Please tick two boxes for each of parts (a) - (d)).

How frequently do pupils in your class:	Pre 5-14 practice				practice/likely practice after implementation of 5-14		
	not at all	very rarely	sometimes	frequently	increase	decrease	about the same
(a) Use calculators?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(b) Estimate answers before using calculators?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(c) Use calculators to check answers arrived at by written means?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(d) Check answers produced on a calculator by doing a second calculation?	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

YOUR USE OF MICROCOMPUTERS FOR MATHEMATICS IN SCHOOL

23. How many microcomputers do you have readily available for use with your class (Please tick the appropriate box)

0	1	2	more than 2
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

24. On average, how long would each pupil, in the space of four weeks, spend working on mathematics on a micro, either individually or in a small group. (Please tick the appropriate box)

less than 30 mins	<input type="checkbox"/>
30-60 mins	<input type="checkbox"/>
1-3 hours	<input type="checkbox"/>
3-5 hours	<input type="checkbox"/>
more than 5 hours	<input type="checkbox"/>

25. This question seeks to identify ways in which your use of software has changed as a result of the implementation of the guidelines. For each of the following kinds of software, please indicate both how often you used it before 5-14 was introduced and how often you will now it after implementation of the guidelines.

	Pre 5-14 practice				practice/likely practice after implementation of 5-14		
	not at all	very rarely	sometimes	frequently	increase	decrease	about the same
(a) turtle graphics packages e.g. logo	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(b) software designed to support specific areas of learning e.g. games, investigations, consolidation and practice programs	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(c) graph drawing packages	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(d) databases	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(e) spreadsheets	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

26. Do you feel that you should make greater use of computers in mathematics lessons than you do at present?

YES ☐

NO ☐

If NO, please go to question 26.

If YES, please tick those reasons why you do not use micros as much as you think you should.

(a) There are too few micros available	<input type="checkbox"/>
(b) I am not sufficiently familiar with the software which is available to me	<input type="checkbox"/>
(c) It is too difficult to organise the use of one machine with a whole class	<input type="checkbox"/>
(d) I feel that I need more in-service training to work well with micros	<input type="checkbox"/>
(e) I find it difficult to fit the available software into the context of ordinary maths lessons	<input type="checkbox"/>

Other reasons (please specify)



YOUR VIEWS ON THE USE OF CONTEXT IN MATHEMATICS

27. How do you feel about the assertion in the guidelines that, "teachers need to identify or create contexts which support what is to be learned and which motivate their pupils" (Tick the box which best matches your views)

- (a) I totally agree with it☐
- (b) I agree but with some reservations☐
- (c) I don't agree with it☐
- (d) I'm not sure that I understand it fully☐
- (e) I agree in theory but find it difficult in practice☐

28. This question seeks to identify changes in your use of context as a result of the implementation of the guidelines. (Please tick two boxes for each of parts (a)-(g)).

	Pre 5-14 practice				practice/likely practice after implementation of 5-14		
	not at all	very rarely	sometimes	frequently	increase	decrease	about the same
(a) managing everyday situations	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(b) designing and making	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(c) studying aspects of the environment	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(d) investigating in science	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(e) investigating areas of mathematics	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(f) play, games and puzzles	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
(g) text book series	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

**YOUR OWN FEELINGS AND ATTAINMENTS IN MATHEMATICS**

**29.** Please tick the highest level you reached in your formal study of mathematics (Do not include mathematics methods classes in a college of education, or other maths education courses)

- (a) O-grade / O-level ☐
- (b) Lower maths ☐
- (c) Standard grade      - credit ☐  
                                     - general ☐  
                                     - foundation ☐
- (d) Higher maths ☐
- (e) Scotvec modules ☐
- (f) Sixth year studies ☐
- (g) A - level ☐
- (h) Degree level ☐
- (i) Other, please specify ☐

**30.** Since qualifying as a teacher, please list any award-bearing courses you have studied in mathematics education e.g. post-graduate certificate, diploma or Open University course.

**31.** In relation to your ability to teach maths at **any** level in the primary school, how would you describe your own mathematical ability? (Please tick one box)

- excellent ☐
- very good ☐
- adequate ☐
- inadequate ☐
- very poor ☐

32. Please show an order of preference for the five curricular areas given. Write a 1 against the area you most prefer to teach followed by a 2 for your second favourite, and so on down to a 5 against the area you least enjoy teaching.

English Language	<input type="checkbox"/>
Environmental Studies	<input type="checkbox"/>
Expressive Arts	<input type="checkbox"/>
Mathematics	<input type="checkbox"/>
Religious & Moral Education	<input type="checkbox"/>

ATTITUDES IN YOUR SCHOOL TO THE GUIDELINES

33. How would you describe your teaching colleagues' reactions to the implementation of the guidelines in your school? (Please tick one box)

positive	<input type="checkbox"/>
neutral	<input type="checkbox"/>
negative	<input type="checkbox"/>
a mixture	<input type="checkbox"/>

34. How influential is the attitude of the school's management team in shaping their teaching staff's attitudes to implementing change? (Please tick one box)

very influential	<input type="checkbox"/>
it has some effect	<input type="checkbox"/>
it has little effect	<input type="checkbox"/>
it has no effect at all	<input type="checkbox"/>

35. What would make you more positive about implementing the guidelines?  
(Please specify)

**AND FINALLY SOME BACKGROUND INFORMATION**

**36.** Class currently taught (e.g. P2, P4/5).....

**37.** Total length of time in teaching

1 year or less ☐

2-3 years ☐

4-10 years ☐

11 years or more ☐

**38.** Are you hoping to retire from teaching within the next 5 years?

YES ☐

NO ☐

**39.** If there is anything you would like to add about your experience of implementing the guidelines, please use this space.

Thank you very much for the time and trouble you have taken to complete this questionnaire.  
Your cooperation is much appreciated and highly valued. Please return it sealed in the envelope provided, if possible by the end of January 1996

Lindsay Logan

November 1996

Problem Solving Record Sheet (Group A schools)

Name: ..... Class .....

Problems			
Name	No.	Code	Date
<div>Teacher's comments</div>			

Schools used in the problem solving study

school I.D.	A or B	rural (R) or urban (U)	size of class	number of participating pupils	stages of participating pupils	type of class composite (C) or not (N/C)
1	A	R	25	7	P7	C
2	A	R	27	2	P7	C
3	A	R	28	17	P5-7	C
4	A	U	30	25	P7	N/C
5	A	U	29	17	P7	N/C
6	A	U	25	14	P7	N/C
7	A	U	24	19	P7	N/C
8	B	R	22	8	P7	C
9	B	U	27	13	P7	N/C
10	B	U	30	27	P7	N/C
11	B	U	28	20	P7	N/C
12	B	U	28	14	P7	N/C
13	B	R	23	10	P7	C
14	B	U	29	19	P7	N/C

Note: The number of participating pupils was the number who wrote both parts of both the pre- and the post-tests.

### Letter to Group A schools

Dear

#### Using the Problem Solving Bank

These notes are to accompany the Problem Solving bank of materials, to explain its purpose and to suggest ways of using it.

#### Folders/Strategies

There are 118 individual problems contained in the six folders. Each of the folders A - F contains problems which may be solved using the same strategy. The strategies related to each folder are as follows:-

A - Look for a pattern;    B - Use Logical reasoning;    C - Guess, Check and Improve the Solution;  
D - Make a Table or an Organised List;  
E - Draw a Picture or a Diagram;    F - Work Backwards.

It is important to realise that these strategies are in a sense quite arbitrary, since many children will use different strategies from those suggested by me. In most cases, a number of strategies will be used to solve the problem. In this sense the strategies which I have attached to each folder are really there as an organising device. The pupils should **not** be given any clue as to the strategy I have attached to each folder. What is important is that the pupils themselves should be aware of the strategy which they used for each problem. They should not associate each folder with any particular strategy. It may well happen that, in light of your experiences in this coming year, you may subsequently wish to re-categorise many of the problems in the folders. This will be perfectly acceptable.

#### Levels of difficulty

Just as the allocation of strategies to the folders is somewhat arbitrary, you may also find my allocation of 'levels' equally unreliable. You will notice that I have classified each problem as either a circle (easy), a square (difficult) or a triangle (somewhere in between). Once again I have done this using my own hunches and in some cases information from the books I have borrowed them from. Please do not put too much reliance on these levels, as I fear that I could be quite mistaken, never having tried most of the problems with P7 children before. Feel free to change the levels in light of your pupils' performance. I will be happy to take them back next June and re-categorise them by strategy and level for following years.

I would suggest that you might wish to start off by giving your pupils some of the easier ones to try first of all, before getting some of your more able ones to try some of the harder problems.

#### Numbering and coding

You will notice that each problem has, in addition to a level, a title and a number. These are intended to help pupils keep a record of each problem as it has been done. You will also note that in the bottom left hand corner of each page there are code numbers and letters. The first letter refers to the letter of the folder to which the problem belongs. The number which appears next identifies the source of the problem. For example 14 refers to the book called *Teaching Problem Solving Strategies*, by Dolan & Williamson, published by Addison Wesley. The attached sheet gives details of all these source books and their reference numbers. The third number, or combination of letters and number, identifies the page or section reference within the book. I must apologise for most of the problems having no illustrations. If time had permitted I would have added many more illustrations to try to make them more attractive. If you would like to alter them in this way yourself, please feel free to do so.

#### Solutions

You will perhaps be disappointed to find no answers. If this creates a difficulty for you I will try to provide a set of solutions.

### **Time allocation**

Could I ask you to allocate about 45 minutes per week to problem solving using this bank of problems, at least for the pupils whom I have identified as being part of my study. For the rest of the pupils in your class, feel free to allocate whatever length of time you see fit. On some occasions your youngsters will manage more than one problem in the time allocated. This is fine. Simply direct them to another one. If the 45 minutes isn't enough time to finish one, use your discretion to allow more time or allow them to take it home.

### **'Teaching'**

Could I ask you to present the problems on a weekly basis as described above for the pupils who will be part of my study. For the rest of your class, please use them as you see fit. You may wish to use a recording format of the type I showed you earlier, in which the children keep a note of their own problem solving activities and a copy of which I have enclosed. You may also wish to attach an extra column to this with a heading such as 'How I solved it'. Please feel free to use or not use this recording format.

If you have any difficulties in implementing my suggestions, or if there are any points which I have not made clear, I shall be happy to come and see you to talk about them, so please don't hesitate to get in touch. In particular it may be that you feel that you need more problems of a particular level. I will try to provide some more if this proves necessary.

Good luck with your problem solving and with the bank of problems which I hope you will find useful. I shall be getting in touch with you early in November to see if I can arrange to have my first chat with your pupils who are part of my study. In the meantime I shall be in Botswana for three and a half weeks returning on November 2nd. Thank you once again for participating in this study.

Yours sincerely

Lindsay Logan



**Teacher Interviews                      June 1996**

Name..... School ..... Date .....

- 1. How many P6/7 pupils will you have next year?
- 2. How many of these would you estimate would be working towards level D in Maths?
- 3. How regularly would you expect your level D pupils to do process problems?
- 4. Which PS resources do you use?
- 5. How are your PS activities organised or structured?
- 6. How much 'teaching' or 'intervention' would you anticipate using?
- 7. To what extent do you make use of the 'Starting/ Doing/ Reporting' cycle when teaching PS?
- 8. Do you teach your pupils about strategies? If so how?
- 9. How do your pupils 'report' their PS activities?
- 10. How many years have you been teaching?.....Teaching PS?.....

### Letter to Group B schools

Dear

#### Using the Problem Solving Bank

These notes are to accompany the Problem Solving bank of materials, to explain its purpose of them and to suggest ways of using it.

#### Folders/Strategies

There are 118 individual problems contained in the six folders. Each of the folders A - F contains problems which may be solved using the same strategy. The strategies related to each folder are as follows:-

A - Look for a pattern;    B - Use Logical reasoning;    C - Guess, Check and Improve the Solution;  
D - Make a Table or an Organised List;  
E - Draw a Picture or a Diagram;    F - Work Backwards.

It is important to realise that these strategies are in a sense quite arbitrary, since many children will use different strategies from those suggested by me. In most cases, a number of strategies will be used to solve the problem. In this sense the strategies which I have attached to each folder are really there as an organising device. The pupils should **not** be given any clue as to the strategy I have attached to each folder. What is important is that the pupils themselves should be aware of the strategy which they used for each problem. They should not associate each folder with any particular strategy. It may well happen that, in light of your experiences in this coming year, you may subsequently wish to re-categorise many of the problems in the folders. This will be perfectly acceptable.

#### Levels of difficulty

Just as the allocation of strategies to the folders is somewhat arbitrary, you may also find my allocation of 'levels' equally unreliable. You will notice that I have classified each problem as either a circle (easy), a square (difficult) or a triangle (somewhere in between). Once again I have done this using my own hunches and in some cases information from the books I have borrowed them from. Please do not put too much reliance on these levels, as I fear that I could be quite mistaken, never having tried most of the problems with P7 children before. Feel free to change the levels in light of your pupils' performance. I will be happy to take them back next June and re-categorise them by strategy and level for following years.

#### Numbering and coding

You will notice that each problem has, in addition to a level, a title and a number. These are intended to help pupils keep a record of each problem as it has been done. You will also note that in the bottom left hand corner of each page there are code numbers and letters. The first letter refers to the letter of the folder to which the problem belongs. The number which appears next identifies the source of the problem. For example 14 refers to the book called *Teaching Problem Solving Strategies*, by Dolan & Williamson, published by Addison Wesley. The attached sheet gives details of all these source books and their reference numbers. The third number, or combination of letters and number, identifies the page or section reference within the book. I must apologise for most of the problems having no illustrations. If time had permitted I would have added many more illustrations to try to make them more attractive. If you would like to alter them in this way yourself, please feel free to do so.

#### Solutions

You will perhaps be disappointed to find no answers. If this creates a difficulty for you I will try to provide a set of solutions.

#### Time allocation

Could I ask you to allocate about 45 minutes per week to problem solving using this bank of problems, at least for the pupils whom I have identified as being part of my study. For the rest of the pupils in your class, feel free to allocate whatever length of time you see fit. On some occasions your youngsters will manage more than one problem in the time allocated. This is fine. Simply direct them to another one. If the 45 minutes isn't enough time to finish one, use your discretion to allow more time or allow them to take it home.

### **'Teaching'**

As you probably know, the 5-14 National Guidelines define a problem solving 'cycle' of 'Starting', 'Doing' and 'Reporting'. For the purposes of my study during this school year, I would like to ask you to think about the 'Reporting' aspect of this cycle. In particular, I would like you to encourage the pupils involved in my study to report on their thoughts, actions and processes when solving problems. Could I suggest that you might wish to do this in the following ways?

1. Give them a recording format which has a column in which they are asked to write how they solved it, or the strategy ( or strategies) used. A sample recording sheet is attached which you may wish to adopt or adapt as you see fit. Whichever recording format you use, could I ask that it contains a column similar to the one just described.
2. Could you try, with the group involved in my study, to chat to them briefly at some time during or just after their problem solving activities to ask them to describe orally their solution strategies or processes. Initially they may need some help in deciding what to write down as a description of how they did it, so you may want to discuss it with them before agreeing what they can write. It is preferable not to tell them how they did it but if possible get them to tell you. If you haven't already done so, it might be helpful to keep a list or a wallchart of strategies which your children have met, used and understood.
3. Do not worry if your pupils give you explanations which you did not expect. There is no definitive list of strategies, nor do strategies have particular names, other than what the children choose to call them.
4. If you do not always have time to get explanations from the children as to how they solved the problem, please ask them either to write it down on their record sheet, or explain their processes to each other. I hope that you will find that with practice they will get better at this and will become more aware of a number of strategies which seem to be useful for getting started and for helping to find solutions.

If you have any difficulties in implementing my suggestions, or if there are any points which I have not made clear, I shall be happy to come and see you to talk about them, so please don't hesitate to get in touch.

Good luck with your problem solving and with the bank of problems which I hope you will find useful. I shall be getting in touch with you early in November to see if I can arrange to have my first chat with your pupils who are part of my study. In the meantime I shall be in Botswana for three and a half weeks returning on November 2nd. Thank you once again for participating in this study.

Yours sincerely

Lindsay Logan



National Survey Data: Cross Tabulations

CROSS TABULATION OF Q1 WITH Q3A

The Guidelines make sensible recommendations

		strongly agree			strongly disagree	
		1	2	3	4	Row Total
Stage of Implementation reached	an early stage of Implementation	1	1	-	-	2 0.6%
		5	40	4	-	49 15.7%
		17	112	1	1	131 41.9%
	Implementation complete	31	92	7	1	131 41.9%
Column Total		54 17.3%	245 78.3%	12 2.8%	2 0.6%	313 100%

CROSS TABULATION OF Q1 WITH Q3B

They are easy to read

		strongly agree			strongly disagree	
		1	2	3	4	Row Total
Stage of Implementation reached	an early stage of Implementation	-	2	-	-	2 0.6%
		2	33	14	1	50 16.2%
		8	89	29	3	129 41.9%
	Implementation complete	14	89	24	-	127 41.2%
Column Total		24 7.8%	213 69.2%	67 21.8%	4 1.3%	308 100%

CROSS TABULATION OF Q1 WITH Q3C

They attempt to relate maths to real life

		strongly agree			strongly disagree	
		1	2	3	4	Row Total
Stage of Implementation reached	an early stage of Implementation	1	-	1	-	2 0.7%
		2	38	10	-	50 16.3%
		10	108	8	2	128 41.7%
	Implementation complete	11	111	5	-	127 41.4%
Column Total		24 7.8%	257 83.7%	24 7.8%	2 0.7%	307 100%

They encourage children to use and apply mathematics to a greater extent than before

		strongly agree		strongly disagree	
		1	2	3	4
Stage of Implementation reached	an early stage of Implementation	1	2	-	-
		-	31	17	1
		14	79	31	3
	Implementation complete	20	72	32	5
	Column Total	34	184	80	9
		11.1%	59.9%	26.1%	2.1%
		2	49	127	129
		0.6%	16%	41.4%	42.0%
		30.7			100%

## CROSS TABULATION OF Q1 WITH Q3E

The targets are realistic

		strongly agree		strongly disagree	
		1	2	3	4
Stage of Implementation reached	An early stage	1	1	1	-
	In progress	2	35	11	-
	Nearly complete	7	107	12	1
	Implementation complete	18	99	14	1
	Column Total	27	242	38	2
		8.7%	78.3%	12.3%	0.6%
		2	48	127	132
		0.6%	15.5%	41.1%	42.7%
		309			100%

## CROSS TABULATION OF Q2 WITH Q6A1

In-service courses delivered by Regional Staff

		Q6A1	had	hadn't	
		1	2		Row Total
Views of the Guidelines	Q2	1	2		
	liked very much	85	39		124
		77	52		129
		20	20		40
		13	9		22
	disliked very much	0	2		2
	Column Total	195	122		317
		61.5%	38.5%		100%

CROSS TABULATION OF Q2 WITH Q6B1

In-service courses delivered by  
College of Education Staff

Views of the Guidelines

Q2	Q6B1	had		hadn't	Row Total
		1	2		
liked very much	1	32	90	122	38.6%
	2	41	90	131	41.5%
	3	8	31	39	12.3%
	4	6	16	22	7.0%
disliked very much	5	0	2	2	0.6%
	Column Total	87	229	316	100%

CROSS TABULATION OF Q2 WITH Q6C1

Advice from Regional advisers or  
other Regional personnel who have  
come to your school

Views of the Guidelines

Q2	Q6C1	had		hadn't	Row Total
		1	2		
liked very much	1	57	66	123	38.9%
	2	54	77	131	41.5%
	3	11	27	38	12.0%
	4	8	14	22	7.0%
disliked very much	5	0	2	2	0.6%
	Column Total	130	186	316	100%

CROSS TABULATION OF Q2 WITH Q6D1

Informal conversations with  
colleagues about the guidelines

Views of the Guidelines

Q2	Q6D1	had		hadn't	Row Total
		1	2		
liked very much	1	118	5	123	38.7%
	2	125	5	130	40.9%
	3	34	6	40	12.6%
	4	22	1	23	7.2%
disliked very much	5	1	1	2	0.6%
	Column Total	300	18	318	100%

CROSS TABULATION OF Q2 WITH Q6E1

Views of the Guidelines

Organised meetings with colleagues from your school				
Q6E1				
had				
hadn't				
Q2		1	2	Row Total
liked very much	1	102	21	123 38.8%
	2	111	19	130 41.0%
	3	32	7	39 12.3%
	4	20	3	23 7.3%
disliked very much	5	1	1	2 0.6%
	Column Total	266 83.9%	51 16.1%	317 100%

CROSS TABULATION OF Q2 WITH Q6F1

Extra time to yourself for reading, planning and thinking e.g. PAT time

		Q6F1	had	hadn't	
Views of the Guidelines	Q 2		1	2	Row Total
	liked very much	1	98	24	122 38.2%
		2	103	28	131 41.1%
		3	29	12	41 12.9%
		4	19	4	23 7.2%
	disliked very much	5	2	0	2 0.6%
		Column Total	251 78.7%	68 21.3%	319 100%

CROSS TABULATION OF Q2 WITH Q6G1

Views of the Guidelines

Advice from your school's promoted staff				
Q6G1				
had				
hadn't				
Q2		1	2	Row Total
liked very much	1	97	26	123 38.7%
	2	100	30	130 40.9%
	3	28	12	40 12.6%
	4	21	2	23 7.2%
disliked very much	5	1	1	2 0.6%
	Column Total	247 77.7%	71 22.3	318 100%



CROSS TABULATION OF Q2 WITH Q6H1

The staff development pack  
"Mathematics (5-14) Exemplification"

Views of the Guidelines

Q2	Q6H1	had		hadn't	Row Total
		1	2		
liked very much	1	54	69	123	38.7
	2	56	75	131	42.2%
	3	14	25	39	12.3%
	4	4	19	23	7.2%
disliked very much	5	0	2	2	0.6%
	Column Total	128	190	318	100%

CROSS TABULATION OF Q2 WITH Q6I1

"5-14 Catalogue: Mathematics" (SCCC  
1993)

Views of the Guidelines

Q2	Q6I1	had		hadn't	Row Total
		1	2		
liked very much	1	50	74	124	38.9%
	2	39	92	131	41.1%
	3	10	29	39	12.2%
	4	4	19	23	7.2%
disliked very much	5	0	2	2	0.6%
	Column Total	103	216	319	100%

CROSS TABULATION OF Q2 WITH Q6J1

"5-14 A Practical Guide" (SOED 1994)

Views of the Guidelines

Q2	Q6J1	had		hadn't	Row Total
		1	2		
liked very much	1	74	47	121	38.4%
	2	60	71	131	41.6%
	3	25	13	38	12.1%
	4	11	12	23	7.3%
disliked very much	5	0	2	2	0.6%
	Column Total	170	145	315	100%

CROSS TABULATION OF Q2 WITH Q6K1

Views of the Guidelines

Printed material produced by your Region e.g. 5-14 Regional Guidelines			
		Q6K1	
		had	hadn't
Q2		1	2
liked very much			Row Total
	1	78	43
	2	68	62
	3	25	14
disliked very much	4	14	9
	5	1	1
	Column Total	186	129
		59%	41%
			315 100%

CROSS TABULATION OF Q2 WITH Q6L1

Views of the Guidelines

Appropriate commercially published material			
		Q6L1	
		had	hadn't
Q2		1	2
liked very much			Row Total
	1	89	32
	2	83	48
	3	22	16
disliked very much	4	14	8
	5	1	1
	Column Total	209	105
		66.6%	33.4%
			314 100%

CROSS TABULATION OF Q36 WITH Q22A1

Use of calculators pre 5-14

Stages taught		Not at all	Very rarely	Sometimes	Frequently	Row Total
		1	2	3	4	
	P1	11	11	6	-	28 14.3%
	P2	3	14	7	-	24 12.2%
	P3	2	7	9	-	18 9.2%
	P4	4	7	10	1	22 11.2%
	P5	3	11	13	-	27 13.8%
	P6	-	10	12	2	24 12.2%
	P7	1	8	28	16	53 27.0%
	Column Total	24 12.2%	68 34.7%	85 43.4%	19 9.7%	196 100%

Number of missing Observations: 132

CROSS TABULATION OF Q36 WITH Q22A2

Use of calculators post 5-14

Stages taught		Increase	Decrease	Same	Row Total
		1	2	3	
	P1	13	-	11	24 12.9%
	P2	12	1	10	23 12.4%
	P3	14	-	3	17 9.1%
	P4	18	-	3	21 11.3%
	P5	19	3	4	26 14.0%
	P6	18	-	5	23 12.4%
	P7	37	-	15	52 28.0%
	Column Total	131 70.4%	4 2.2%	51 27.4%	186 100%

Number of missing Observations: 150

CROSS TABULATION OF Q36 WITH Q25D1

Use of databases pre 5-14

Stages taught

	Not at all	Very rarely	Sometimes	frequently	
	1	2	3	4	Row Total
P 1	22	2	2	-	26 13.8%
P 2	14	4	4	1	23 12.2%
P 3	10	6	2	-	18 9.5%
P 4	11	5	3	1	20 10.6%
P 5	8	6	12	-	26 13.8%
P 6	9	7	7	-	23 12.2%
P 7	23	14	15	1	53 28.0%
Column Total	97 51.3%	44 23.3%	45 23.8%	3 1.6%	189 100%

Number of missing Observations: 139

CROSS TABULATION OF Q36 WITH Q25D2

Use of databases post 5-14

Stages taught

	Increase	Decrease	Same	
	1	2	3	Row Total
P 1	10	-	12	22 12.4%
P 2	4	-	14	18 10.1
P 3	7	-	11	18 10.1%
P 4	15	-	6	21 11.8%
P 5	16	2	7	25 14.0%
P 6	12	-	10	22 12.4%
P 7	36	-	16	52 29.2%
Column Total	100 56.2%	2 1.1%	76 42.7%	178 100%

Number of missing Observations: 15

CROSS TABULATION OF Q36 WITH Q28G1

Textbook usage pre 5-14

Stages taught

	Not at all	Very rarely	Sometimes	Frequently	Row Total
	1	2	3	4	
P1	2	1	9	17	29 15.4%
P2	4	3	7	10	24 12.8%
P3	-	-	9	9	18 9.6%
P4	-	1	2	19	22 11.7%
P5	1	-	6	19	26 13.8%
P6	-	2	6	13	21 11.2%
P7	1	1	9	37	48 25.5%
Column Total	8 4.3%	8 4.3%	48 25.5%	124 66.0%	188 100%

Number of missing Observations: 140

CROSS TABULATION OF Q36 WITH Q28G2

Textbook usage post 5-14

Stages taught

	Increase	Decrease	Same	Row Total
	1	2	3	
P1	3	4	22	29 15.5%
P2	4	5	15	24 12.8%
P3	4	4	11	19 10.2%
P4	2	7	13	22 11.8%
P5	2	5	14	21 10.7%
P6	5	4	11	20 10.7%
P7	8	9	31	48 25.7%
Column Total	28 15.3%	38 20.7%	117 63.9%	183 100%

Number of missing Observations: 141

CROSS TABULATION OF Q31 WITH Q32D

	Most preferred subject				Least preferred subject	
	1	2	3	4	5	Row Total
1	23	7	2	2	1	35 11.2%
2	52	55	27	13	2	149 47.6%
3	14	54	33	16	4	121 38.7%
4	-	1	-	5	1	7 2.2%
5	-	-	-	-	1	1 0.3%
Column Total	89 28.4%	117 37.4%	62 19.8%	36 11.5%	9 2.9%	313 100%

CROSS TABULATION OF Q31 WITH Q29

Highest Qualification attained in mathematics

		O-grade	Lower Maths	Standard Grade Maths	Higher Maths	Scotvec Modules	CSYS	A-Level	Degree	Other		
Teachers' ratings of their ability to teach maths at any level in the primary school		1	2	3	4	5	6	7	8	9	Row Total	
	Excellent	1	9	2	-	18	-	1	2	3	1	36 11.4%
	Very good	2	67	7	-	71	-	2	2	2	3	154 48.7%
	Adequate	3	80	10	-	24	2	-	-	-	2	118 37.3%
	Inadequate	4	3	-	2	2	-	-	-	-	-	7 2.2%
	Very poor	5	1	-	-	-	-	-	-	-	-	1 0.3%
	Column Total		160 50.6%	19 6%	2 0.6%	115 36.4%	2 0.6%	3 0.9%	4 1.3%	5 1.6%	6 1.9%	316 100%

CROSS TABULATION OF Q32D WITH Q29

Highest qualifications gained in Mathematics

		O-grade	Lower Maths	Standard Grade Maths	Scotvec Modules	Higher Maths	CSYS	A-Level	Degree	Other	
		1	2	3	5	4	6	7	8	9	Row Total
Most preferred teaching subject	1	34	2	-	-	42	3	2	4	2	89 29.8%
	2	59	12	-	1	42	-	-	-	2	116 37.5%
	3	35	5	-	-	17	-	-	1	1	59 19.1%
	4	24	-	2	1	7	-	2	-	-	36 11.7%
Least preferred teaching subject	5	6	-	-	-	3	-	-	-	-	9 2.9%
	Column Total	158 51.1%	19 6.1%	2 0.6%	2 0.6%	111 35.9%	3 1.0%	4 1.3%	5 1.6%	5 1.6%	309 100%

CROSS TABULATION OF Q2 WITH Q37

Length of time in teaching

		Less than one year				2-3 years		4-10 years		11 years of more		
		1	2	3	4	5	6	7	8	9	10	Row Total
Teacher views of the guidelines	Like very much	P 1	2	5	19	95	121 38.7%					
		P 2	-	6	24	98	128 40.0%					
		P 3	-	2	7	31	40 12.8%					
		P 4	-	-	3	19	22 7.0%					
	Dislike very much	P 5	-	-	-	2	2 0.6%					
	Column Total	2 0.6%	13 4.2%	53 16.9%	245 78.3%	313 100%						

National Survey Data: Correlations

CORRELATION COEFFICIENTS – Q2 WITH Q3A-E

	Q3A	Q3B	Q3C	Q3D	Q3E
Q2	0.5051**	0.4763**	0.2449**	0.4134**	0.4296**

\* - Signif. LE .05    \*\* - signif. LE.01    (2-tailed)

CORRELATION COEFFICIENTS – Q9A-F WITH Q12A-F

	Q9A	Q9B	Q9C	Q9D	Q9E	Q9F
Q12A	-0.12*	0.23**	0.15**	-0.10	0.25**	-0.13*
Q12B	-0.10	0.18**	0.04	0.04	0.13*	0.00
Q12C	-0.86	0.13*	0.04	0.03	0.12*	0.00
Q12D	-0.16**	0.22**	-0.10	-0.01	0.18**	-0.06
Q12E	-0.07	0.20**	-0.09	0.05	0.17**	-0.04
Q12F	-0.05	0.18**	0.01	0.02	0.16**	0.04

\* - Signif. LE .05    \*\* - signif. LE.01    (2-tailed)



The Pre-test

?? ? PROBLEMS 1 ?? ?

Pre Test

Name: \_\_\_\_\_

On this and the next few pages there are a number of problems for you to solve.

Try to do each one on your own and **do all your workings, or drawings or diagrams, in the space provided below each question.**

When you find the answer make sure it is clearly shown. If you can't do one, try the next one.

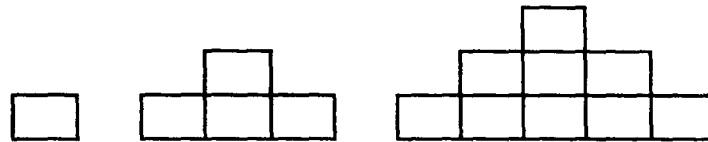
1. POCKET MONEY



If the person with most money had 17p, how much did each one have?

2. WALL OF CHINA

Here are the first three shapes in a sequence.



How many squares are needed to make:

(a) the fifth shape .....?

(b) the tenth shape .....?

Explain your answer.

3.

### PICTURE FRAMES

The picture shown on the right measures 3cm x 3cm and is framed by a number of 1cm tiles.

How many of these tiles would you need to frame a 4cm x 4cm picture?

A 5cm x 5cm picture?

Can you find and explain a way of finding how many tiles would be needed to frame a picture which measured 27cm x 27 cm?



## Appendix 6.1

## The Pre-test

## PROBLEMS 2

### Pre Test

**Name:** \_\_\_\_\_

On this and the next few pages there are a number of problems for you to solve.

Try to do each one on your own and **do all your workings, or drawings or diagrams, in the space provided below each question.**

When you find the answer make sure it is clearly shown. If you can't do one, try the next one.

4.

## HALF-TIME SCORES

This was the final score of a football match some years ago.

**MONTROSE - 4      BRECHIN - 3**

### What were the possible scores at half-time?

**5. THE TEACHERS' LUNCH**

Five teachers are having lunch together seated round a circular table.

Miss Vincent is sitting between Miss Russell and Mrs Wilson.

Betty is sitting with Mrs Taylor on her left and Mrs Wilson on her right.

Cathy is sitting between Amy and Mrs Smith.

Miss Russell is sitting between Cathy and Debbie.

One teacher's first name is Anne.

**Can you match first names to second names and show who is sitting next to whom?**

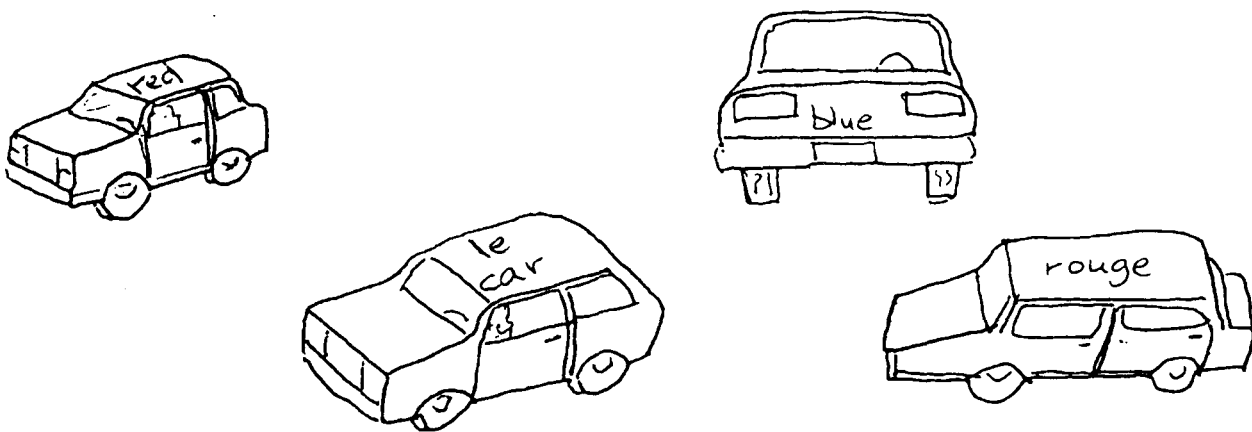
6. THE MOTOR SHOW

Every red car at a motor show was a French car.

Half of all the blue cars were French.

Half of all the French cars were red.

There were 40 blue cars and 30 red cars.



How many French cars were neither blue or red?

Explain your answer.

# The Post-test

## ??? PROBLEMS 1 ???

Post Test

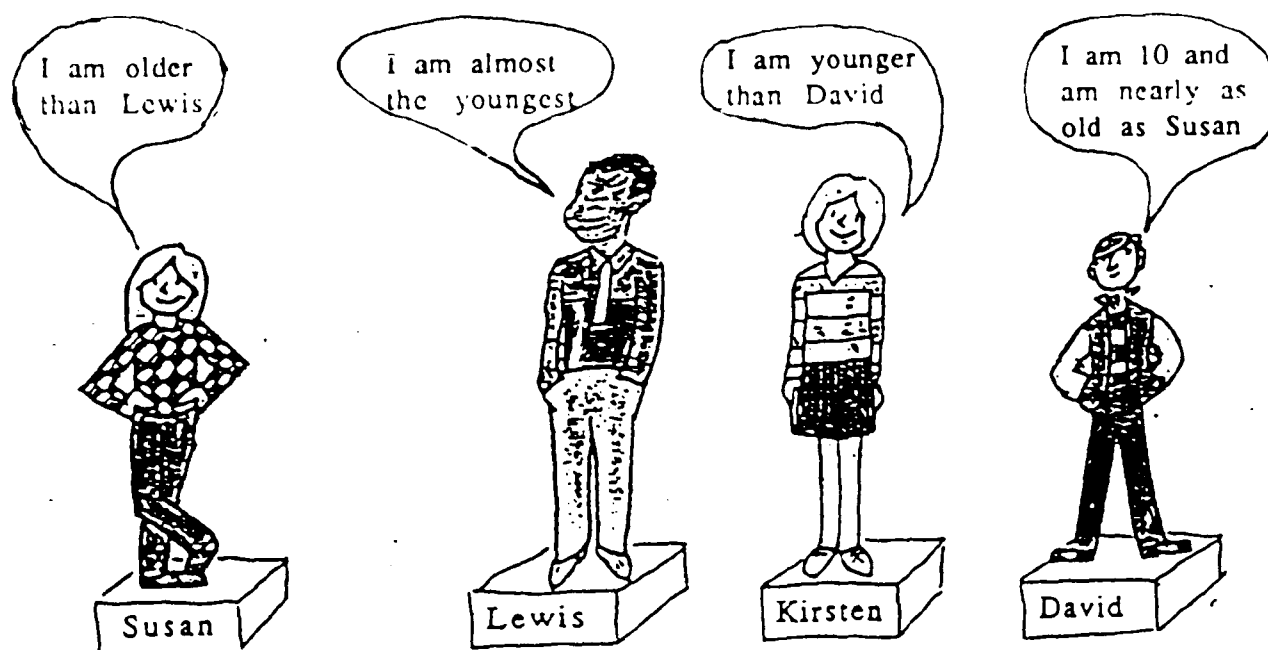
Name: \_\_\_\_\_

On this and the next two pages there are three problems for you to try and solve.

Try to do each one on your own and **do all your workings, or drawings or diagrams, in the space provided below each question.**

When you find the answer make sure it is clearly shown. If you can't do one, try the next one.

### 1. OUR AGES



Put the children's names in order of their ages, youngest to the left and oldest to the right.

Youngest \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, Oldest

2.

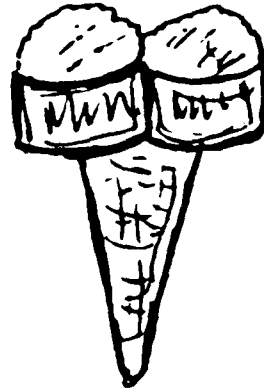
## ICE CREAM FLAVOURS

Mhairi went to the ice cream van to buy a 'double scoop' cone for her little brother.

He wanted the two scoops to be different flavours.

The ice cream van had four flavours – vanilla, orange, strawberry and mint.

How many different cones could Mhairi choose from and what were their flavours.





3. COINS IN A BAG

Philip's grandad gave him £1.35 to spend.

The money consisted of 10 coins in a plastic bag.

There were some 5p, some 10p and some 20p coins.

How many of each kind of coin did Philip get?

Appendix 6.2

The Post-test

?

?

?

PROBLEMS 2

?

?

?

Post Test

Name: \_\_\_\_\_

On this and the next two pages there are three problems for you to try and solve.

Try to do each one on your own and **do all your workings, or drawings or diagrams, in the space provided below each question.**

When you find the answer make sure it is clearly shown. If you can't do one, try the next one.

4.

DINOSAUR BUTTONS



Emma and Jemma had a bag with 25 buttons with dinosaur pictures on them.

Each button had either a Tyrannosaurus or a Diplodocus on it.

There were 7 more Tyrannosaurus buttons than Diplodocus ones.

How many of each kind of button were there?

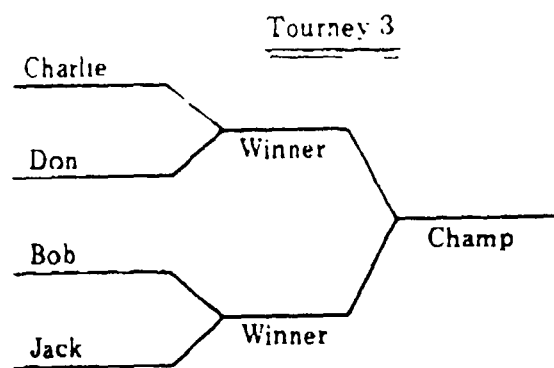
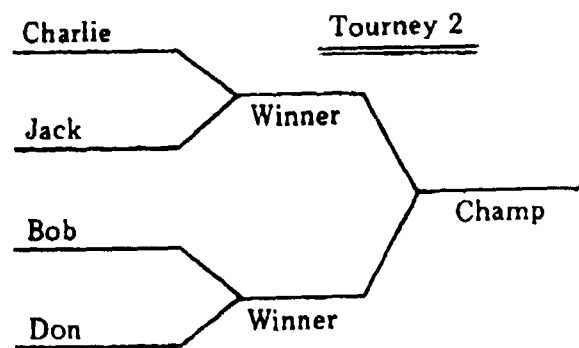
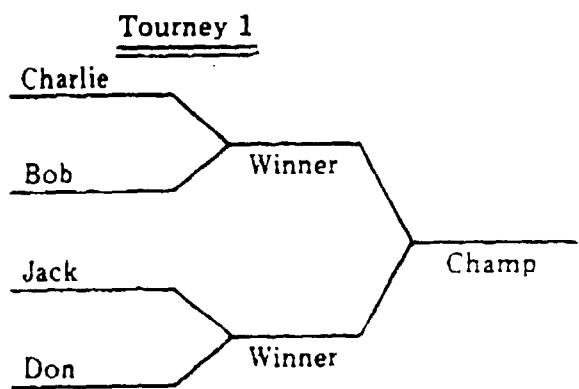
5. PING-PONG CHAMPS

Charlie, Bob, Jack and Don held three ping-pong tournaments.

Bob never defeated Don. Charlie lost all of his games.

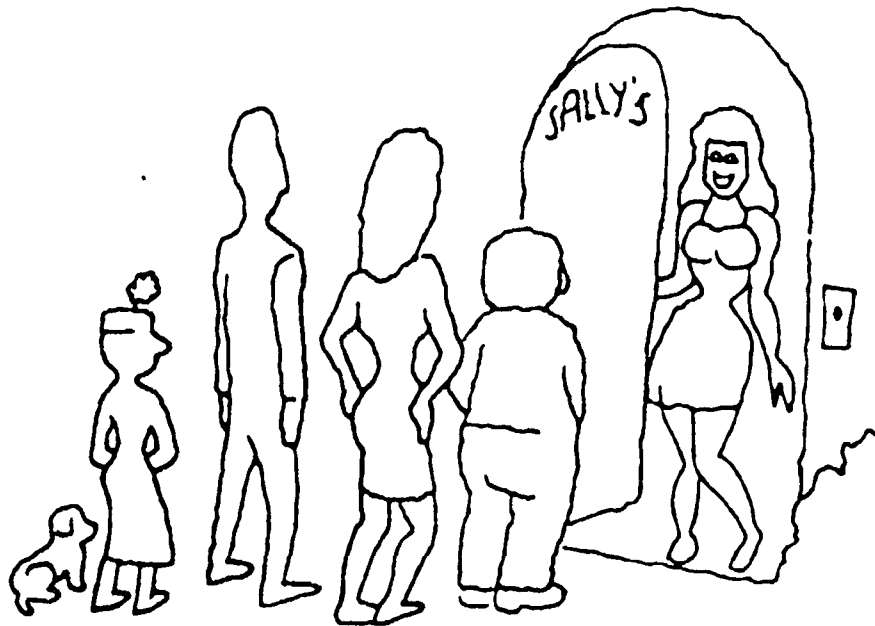
Don never defeated Jack. Only Charlie did not win a tournament.

Can you complete the three sets of tournament results below?



6.

SALLY'S PARTY



Sally is having a party. The first time her doorbell rings, 1 guest comes in.

The second time it rings, 3 guests come in.

Each time the bell rings 2 more guests come in than the time before.

- a) How many people enter on the sixth ring?
- b) How many guests will be there altogether after 6 rings?
- c) Can you say how many guests would there be after 10 rings?
- d) Can you describe a pattern for the total number of guests after each ring?